PART – A

Answer any TEN questions 10 X1=10

1. Give an example of a relation which is symmetric and transitive but not reflexive.

2. Define a binary operation.

3. Find the principal value of $\cos^{-1}\left(-\frac{1}{2}\right)$.

4. If $\sin\left(\sin^{-1}\frac{1}{5} + \cos^{-1} x\right) = 1$, then find the value of $x$.

5. Define a row matrix.

6. Find the value of $x$ if $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$.

7. If $y = e^{\cos x}$, find $\frac{dy}{dx}$.

8. If $y = \sin(x^2 + 5)$, find $\frac{dy}{dx}$.

9. Find $\int (2x^2 + e^x)\,dx$.

10. Evaluate $\int_{2}^{3} \frac{1}{x^2}\,dx$.

11. Find the unit vector in the direction of vector $\vec{a} = 2\hat{i} + 3\hat{j} + \hat{k}$.
12. Write two different vectors having same magnitude.

13. Write the direction cosines of x-axis.


15. Find $P(A|B)$, if $P(B) = 0.5$ and $P(A \cap B) = 0.32$.

**PART- B**

Answer any TEN questions $10 \times 2 = 20$

16. Show that the signum function $f: \mathbb{R} \rightarrow \mathbb{R}$ given by

$$f(x) = \begin{cases} 
1 & \text{if } x > 0 \\
0 & \text{if } x = 0 \\
-1 & \text{if } x < 0
\end{cases}$$

is neither one-one nor onto.

17. Find the value of $\tan^{-1} \sqrt{3} - \sec^{-1}(-2)$.

18. Write the domain and range of $y = \tan^{-1} x$.

19. Find the values of $x$, $y$ and $z$ from the equation

$$\begin{bmatrix} 4 & 3 \\ x & 5 \end{bmatrix} = \begin{bmatrix} y \\ z \end{bmatrix}.$$

20. Find equation of line joining $(1, 2)$ and $(3, 6)$ using determinants.

21. If $x^2 + xy + y^2 = 100$, find $\frac{dy}{dx}$.

22. If $x = at^2$, $y = 2at$, find $\frac{dy}{dx}$.

23. Differentiate $\sin(\cos(x^2))$ with respect to $x$.

24. Find the slope of tangent to curve $y = x^3 - x + 1$ at the point whose x-co-ordinate is 2.

25. Find $\int \frac{(\log x)^2}{x} \, dx$.

26. Find $\int \frac{\sin^2 x - \cos^2 x}{\sin^2 x \cos^2 x} \, dx$.

27. Evaluate $\int_{0}^{\frac{\pi}{2}} \cos^2 x \, dx$.

28. Find the order and degree of the differential equation $y' + y = e^x$.

29. Find the projection of the vector $\vec{a} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ on the vector $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$. 
30. Find the area of the parallelogram whose adjacent sides are given by the vectors 
\[ \vec{a} = \hat{i} - \hat{j} + 3\hat{k} \text{ and } \vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}. \]

31. Find the intercepts cut-off by the plane \(2x + y - z = 5\).

32. Find the distance of the point \((-6,0,0)\) from the plane \(2x - 3y + 6z - 2 = 0\).

33. The random variable \(X\) has a probability distribution \(P(X)\) of the following form where \(k\) is some number. Find the value of \(k\).

\[
P(X) = \begin{cases} 
  k, & \text{if } x = 0 \\
  2k, & \text{if } x = 1 \\
  3k, & \text{if } x = 2 \\
  0, & \text{otherwise}
\end{cases}
\]

PART – C

Answer any TEN questions 10 \(X3=30\)

34. Show that the relation \(R\) in the set \(\{1, 2, 3\}\) given by \(R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3)\}\) is reflexive but neither symmetric nor transitive.

35. Prove that \(2 \tan^{-1} \frac{1}{2} + \tan^{-1} \frac{1}{7} = \tan^{-1} \frac{31}{17}\).

36. Find the inverse of the matrix \(A = \begin{bmatrix} 2 & 3 \\
 5 & 7 \end{bmatrix}\) using elementary operations.

37. Verify that the value of the determinant remains unchanged if its rows and columns are interchanged by considering third order determinant \(\begin{vmatrix} 2 & -3 & 5 \\
 6 & 0 & 4 \\
 1 & 5 & -7 \end{vmatrix}\).

38. If \(xy = e^{x-y}\) find \(\frac{dy}{dx}\).

39. If \(x = a(\cos \theta + \theta \sin \theta), y = a(\sin \theta - \theta \cos \theta)\) find \(\frac{dy}{dx}\).

40. Verify mean value theorem, if \(f(x) = x^2 - 4x - 3\) in the interval \([a, b]\)

where \(a = 1\) and \(b = 4\).

41. Find the intervals in which the function \(f\) given by \(f(x) = 2x^3 - 3x^2 - 36x + 7\) is
(a) increasing  (b) decreasing.

42. Find \( \int \frac{xe^x}{(1+x)^2} \, dx \).

43. Evaluate \( \int \frac{1}{(x+1)(x+2)} \, dx \).

44. Evaluate \( \int_0^2 e^x \, dx \) as the limit of a sum.

45. Find the area of the region bounded by \( y^2 = 9x \), \( x = 2 \), \( x = 4 \) and the x-axis in the first quadrant.

46. Form the differential equation representing the family of curves \( y = a \cdot \sin(x+b) \), where \( a, b \) are arbitrary constants.

47. Find the general solution of the differential equation \( \frac{dy}{dx} = \frac{1+y^2}{1+x^2} \).

48. If \( \vec{a}, \vec{b} \) and \( \vec{c} \) are unit vectors such that \( \vec{a} + \vec{b} + \vec{c} = \vec{0} \), find the value of \( \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a} \).

49. Find \( x \) such that the four points \( A(3,2,1), B(4,x,5), C(4,2,-2) \) and \( D(6,5,-1) \) are co-planar.

50. Find the shortest distance between the lines \( \frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1} \) and \( \frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1} \).

51. A man is known to speak truth 3 out of 4 times. He throws a die and reports that it is a six. Find the probability that it is actually a six.

**PART –D**

**Answer any SIX questions**

52. Let \( A = \mathbb{R} - \{3\} \) and \( B = \mathbb{R} - \{1\} \). Consider the function \( f: A \rightarrow B \) defined as \( f(x) = \frac{x-2}{x-3} \). Is \( f \) one-one and onto? Justify your answer.

53. Let \( f: \mathbb{N} \rightarrow \mathbb{R} \) be a function defined as \( f(x) = 4x^2 + 12x + 15 \).

Show that \( f: \mathbb{N} \rightarrow S \) where, \( S \) is the range of \( f \) is invertible. Find the inverse of \( f \).
54. If \( A = \begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix} \) and \( B = \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix} \), then verify that

(i) \((A+B)^T = A^T + B^T\)  \hspace{1cm} (ii) \((A-B)^T = A^T - B^T\)

55. Solve system of linear equation, using matrix method.

\[
\begin{align*}
2x + 3y + 3z &= 5 \\
x - 2y + z &= -4 \\
3x - y - 2z &= 3
\end{align*}
\]

56. If \( y = 3 \cos(logx) + 4 \sin(logx) \) Show that \( x^2 y_2 + xy_1 + y = 0 \).

57. The length \( x \) of a rectangle is decreasing at the rate of 5cm/minute and the width \( y \) is increasing at the rate of 4cm/minute. When \( x = 8 \) cm and \( y = 6 \) cm find the rates of change of

a) the perimeter 

b) the area of the rectangle.

58. Find the integral of \( \frac{1}{x^2 + a^2} \) with respect to \( x \) and hence evaluate \( \int \frac{3x^2}{x^6 + 1} \) \( dx \).

59. Find the area enclosed by the circle \( x^2 + y^2 = a^2 \) using integration.

60. Find the general solution of the differential equation \( x \frac{dy}{dx} + 2y = x^2 \ (x \neq 0) \).

61. Derive the equation of a plane perpendicular to a given vector and passing through a given point both in vector and Cartesian form.

62. A person buys a lottery ticket in 50 lotteries, in each of which his chance of winning a prize is \( \frac{1}{100} \). What is the probability that he will win a prize

a) at least once 

b) exactly once 

63. Probability of solving specific problem independently by A and B are \( \frac{1}{2} \) and \( \frac{1}{3} \) respectively.

If both try to solve the problem independently, find the probability that

(i) the problem is solved \hspace{1cm} (ii) exactly one of them solves the problem
PART – E

Answer any ONE question1 X10=10

64. (a) Maximise $Z = 3x + 2y$ subject to the constraints $x + 2y \leq 10$, $3x + y \leq 15$, $x,y \geq 0$

   b) If the matrix $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ satisfies the equation $A^2 - 4A + I = O$, when $I$ is $2 \times 2$

       identify matrix and O is $2 \times 2$ zero matrix. Using this equation find $A^{-1}$.

65. (a) Prove that $\int_0^a f(x)dx = \int_0^a f(a - x)dx$ and hence evaluate $\int_0^a \frac{\sqrt{x}}{\sqrt{x} + \sqrt{a-x}}dx$.

   (b) Find the value of $K$ so that the function $f(x) = \begin{cases} Kx + 1 & \text{if } x \leq 5 \\ 3x - 5 & \text{if } x > 5 \end{cases}$

       is continuous at $x = 5$.

66. (a) Prove that the volume of the largest cone that can be inscribed in a sphere of

       radius $R$ is $\frac{4}{27}$ of the volume of the sphere.

   b) By using properties of determinants

       Show that $\begin{vmatrix} a - b - c & 2a & 2a \\ 2b & b - c - a & 2b \\ 2c & 2c & c - a - b \end{vmatrix} = (a + b + c)^3$.

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