Model Question Paper – March 2016

II P.U.C MATHEMATICS (35)

Time : 3 hours 15 minute **Max. Marks : 100**

Instructions :

(i) The question paper has five parts namely A, B, C, D and E. Answer all the parts.
(ii) Use the graph sheet for the question on Linear programming in PART E.

PART – A

Answer ALL the questions: **$10 \times 1 = 10$**

1. Find the number of all one-one functions from set $A = \{1,2,3\}$ to itself.
2. Write the range of $f(x) = \sin^{-1} x$ in $[0, 2\pi]$ other than $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.
3. If A is a square matrix of order 2 and $A^{-1} = \frac{\text{adj}A}{10}$, then find $|3A|$.
4. If the matrix $\begin{bmatrix} 5-x & 2y-8 \\ 2 & 3 \end{bmatrix}$ is a symmetric matrix, find the values of x and y.
5. Differentiate $e^{\log_e x}$, $x > 0$, with respect to x.
6. Evaluate: $\int \sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)dx$.
7. For what value of λ, the vectors $\vec{a} = 2\hat{i} - 3\lambda\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + \hat{j} - 2\hat{k}$ are perpendicular to each other?
8. Find the direction ratios of the line $2x = \frac{1-y}{2} = \frac{z+4}{6}$.
9. Define constraints in Linear Programming Problem.
10. If P(A)=0.3, P(not B) = 0.4 and A and B are independent events, find P(A and not B).

PART B

Answer any TEN questions: **$10 \times 2 = 20$**

11. A binary operation * on the set {1,2,3,4,5} is defined by a*b=min{a, b}, write the operation table for the operation *.
12. Simplify: $\tan^{-1}\left[\frac{a\cos x - b\sin x}{b\cos x + a\sin x}\right]$, if $\frac{a}{b}\tan x > -1$.
13. If $\sqrt{x} + \sqrt{y} = \sqrt{5}$, prove that $\frac{dy}{dx} = -\frac{3}{2}$ when $x = 4$ and $y = 9$.
14. If the matrix $A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ and $A^2 = kA$, then write the value of k.

15. Differentiate $(x^2-5x+8)(x^2+7x+9)$ with respect to x, by logarithmic differentiation.

16. Find $\int_2^3 \frac{x}{x^2+1}dx$.

17. Find $\int \frac{dx}{(x+1)(x+2)}$.

18. Form the differential equation of the family of parabolas having vertex at origin and axis along positive y-axis.

19. For any two vectors $\vec{a}$ and $\vec{b}$, prove that $|\vec{a}\cdot\vec{b}| \le |\vec{a}||\vec{b}|$.

20. Let the vectors $\vec{a}$ *and* $\vec{b}$ be such that $|\vec{a}| = 3$, $|\vec{b}| = \frac{\sqrt{2}}{3}$, and $\vec{a}\times\vec{b}$ is a unit vector. Find the angle between $\vec{a}$ *and* $\vec{b}$.

21. Show that the equation of the tangent to the parabola $y^2 = 4ax$ at $(at^2, 2at)$ is $x - yt + at^2 = 0$.

22. Find the coordinates of the point where the line through the points $A(3,4,1)$ and $B(5,1,6)$ crosses XY - plane.

23. Solve the equation: $\sin^{-1}(1-x) = \frac{\pi}{2} + 2\sin^{-1}x$.

24. Find the probability distribution of number of heads in two tosses of a coin.

PART C

Answer any TEN questions: **10 × 3=30**

25. Let Z be the set of all integers and R is the relation on Z defined as R={(a, b): a ,b ∈Z and a-b is divisible by 5}. Prove that R is an equivalence relation.

26. Prove that $\tan^{-1}\left(\frac{1}{2}\right)+\tan^{-1}\left(\frac{1}{5}\right)+\tan^{-1}\left(\frac{1}{8}\right)=\frac{\pi}{4}$.

27. Using elementary transformations, find the inverse of the matrix $A=\begin{bmatrix}1 & 3\\ 2 & 7\end{bmatrix}$.

28. Verify Mean Value theorem, if $f(x)=x^3-5x^2-3x$, in the interval [1,3]. Find all $c \in (1,3)$ for which $f'(c) = 0$.

29. If $y=\sin^{-1}\left(\frac{2^{x+1}}{1+4^x}\right)$, find $\frac{dy}{dx}$.

30. Find local maximum and local minimum values of the function f given by $f(x)=3x^4+4x^3-12x^2+12$.

31. The scalar product of the vector $\hat{\imath}+\hat{\jmath}+\hat{k}$ with unit vector along the sum of vectors $2\hat{\imath}+4\hat{\jmath}-5\hat{k}$ and $\lambda\hat{\imath}+2\hat{\jmath}+3\hat{k}$ is equal to one. Find the value of λ.

32. Find $\int e^x \sin x\, dx$.

33. Find the equation of the plane that contains the point $(1,-1,2)$ and is perpendicular to each of the planes $2x+3y-2z=5$ and $x+2y-3z=8$.

34. Show that the vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ are coplanar, if $\vec{a}+\vec{b}$, $\vec{b}+\vec{c}$ and $\vec{c}+\vec{a}$ are coplanar.

35. Evaluate: $\int \frac{1}{1+\tan x}dx$.

36. Find the equation of the curve passing through the point $(1,1)$ whose differential equation is $xdy=(2x^2+1)dx \ (x \neq 0)$.

37. Find the area of the region bounded by the curve $y=x^2+2$, $y=x$, $x=0$ and $x=3$.

38. A bag contains 4 red and 4 black balls, another bag contains 2 red and 6 black balls. One of the two bags is selected at random and a ball is drawn from the bag which is found to be red. Find the probability that the ball is drawn from the first bag.

PART D

Answer any SIX questions: **$6 \times 5 = 30$**

39. Let $A = \ -\{3\}$ and $B = \ -\{1\}$. Consider the function $f: A \to B$ defined by $f(x)=\frac{x-2}{x-3}$. Show that f is invertible and write the inverse of f.

40. If $A=\begin{bmatrix}1 & 1 & -1\\ 2 & 0 & 3\\ 3 & -1 & 2\end{bmatrix}$, $B=\begin{bmatrix}1 & 3\\ 0 & 2\\ -1 & 4\end{bmatrix}$ and $C=\begin{bmatrix}1 & 2 & 3 & -4\\ 2 & 0 & -2 & 1\end{bmatrix}$ then find A(BC) and (AB)C. Show that A(BC)=(AB)C.

41. A water tank has the shape of an inverted right circular cone with its axis vertical and vertex lowermost. Its semi vertical angle is $\tan^{-1}(0.5)$. Water is poured into it at a constant rate of 5 cubic metre per hour. Find the rate at which the level of the water is rising at the instant when the depth of water in the tank is 4 m.

42. If $x=a\left(\cos\theta+\log\tan\frac{\theta}{2}\right)$ and $y=a\sin\theta$, show that $\frac{dy}{dx}=\tan\theta$ and $\frac{d^2y}{dx^2}=\frac{1}{a}\frac{\sec^3\theta}{\sin\theta}$

43. If $A=\begin{bmatrix}2 & -3 & 5\\ 3 & 2 & -4\\ 1 & 1 & -2\end{bmatrix}$, find A^{-1}. Using A^{-1} solve the system of equations $2x-3y+5z=11$; $3x+2y-4z=-5$ and $x+y-2z=-3$.

44. Find the particular solution of the differential equation $\frac{dy}{dx}+y\cot x=4x\cdot \operatorname{cosec} x$, $x \neq 0$, given that $y=0$ when $x=\frac{\pi}{2}$.

45. Find the integral of $\frac{1}{\sqrt{a^2-x^2}}$ with respect to x and hence evaluate $\int\frac{dx}{\sqrt{7-6x-x^2}}$.

46. Derive a formula to find the shortest distance between the two skew line $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu\vec{b_2}$ in the vector form.

47. Find the area of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, $(a > b)$ by the method of integration and hence find the area of the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$.

48. Five cards are drawn successively with replacement from a well shuffled deck of 52 playing cards. What is the probability that
   (i) all the five cards are spades?
   (ii) only 3 cards are spades?
   (iii) none is a spade?

PART E

Answer any ONE question **$1 \times 10=10$**

49. (a) Prove that $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$ and hence evaluate $\int_{-1}^{2}|x^3 - x|dx$. 6

(b) Prove that $\begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix} = (1-x^3)^2$. 4

50. (a) A cooperative society of farmers has 50 hectare of land to grow two crops X and Y. The profit from crops X and Y per hectare are estimated as Rs 10,500 and Rs 9,000 respectively. To control weeds, a liquid herbicide has to be used for crops X and Y at rates of 20 litres and 10 litres per hectare. Further no more than 800 litres of herbicide should be used in order to protect fish and wild life using a pond which collects drainage from this land. How much should be allocated to each crop so as to maximize the total profit of the society? 6

(b) If the function $f(x) = \begin{cases} 3ax+b, & \text{if } x>1 \\ 11, & \text{if } x=1 \\ 5ax-2b, & \text{if } x<1 \end{cases}$ is continuous at $x = 1$, find the values of a and b. 4

Note: It is only a pattern of question paper.