

MODEL QUESTION PAPER FOR SUMMATIVE ASSESSMENT

I P.U.C MATHEMATICS (35)

Time : 3 hours 15 minutes

Max. Marks : 90

Instructions: *The question paper has five parts namely A, B, C, D, E. Answer all the parts.*

PART – A

I Answer any TEN questions: 10 × 1 = 10

1. Given that the number of subsets of a set A is 16. Find the number of elements in A.
2. If $\tan x = \frac{3}{4}$ and x lies in the third quadrant, find $\sin x$.
3. Find 'n' if ${}^n C_7 = {}^n C_6$.
4. Solve $7x + 3 < 5x + 9$, $x \in \mathbb{N}$.
5. The arithmetic mean of 4 and another number is 10. Find the other number.
6. Find the distance between $3x + 4y + 5 = 0$ and $6x + 8y + 2 = 0$.
7. Given $f(x) = \begin{cases} \frac{x}{|x|}, & x \neq 0 \\ 2, & x = 0 \end{cases}$, find $\lim_{x \rightarrow 0^+} f(x)$.
8. Write the negation of 'For all $a, b \in I$, $a - b \in I$ '.
9. If the coefficient of variation and standard deviation are 60, 21 respectively, what is the arithmetic mean of the distribution?
10. Name the octant in which $(4, 2, -5)$ lies?
11. Find the eccentricity of $\frac{x^2}{9} - \frac{y^2}{16} = 1$.
12. Let $A = \{2, 3, 4\}$ and R be a relation on A defined by $R = \{(x, y) \mid x, y \in A, x \text{ divides } y\}$, find 'R'.

PART – B

II Answer any TEN questions: 10 × 2 = 20

13. If A and B are two disjoint sets and $n(A) = 15$ and $n(B) = 10$ find $n(A \cup B)$, $n(A \cap B)$.
14. $f : \mathbb{Z} \rightarrow \mathbb{Z}$ is a linear function defined by $f = \{(1,1), (0,-1), (2,3)\}$, find $f(x)$.

15. The minute hand of a clock is 2.1 cm long. How far does its tip move in 20 minutes? $\left(\text{use } \pi = \frac{22}{7}\right)$.
16. Find the general solution of $2\cos^2 x - 3\sin x = 0$.
17. Prove that $\frac{\sin x + \sin 3x}{\cos x + \cos 3x} = \tan 2x$.
18. Find n such that ${}^n P_5 = 42 {}^n P_3$.

OR

- If the sum of a certain number of terms of the A.P 25, 22, 19 . . . is 116 find the number of terms.
19. Write the inverse, converse of 'If a parallelogram is a square then it is a rhombus'.
20. Find the term independent of x in the expansion of $\left(x^2 + \frac{1}{x}\right)^9$.
21. Find the number of permutations of the letters of the word MISSISSIPPI in which all 4 S's are together and 2 P's are together.
22. Let $z_1 = 2 - i$, $z_2 = -2 + i$. Find the imaginary part of $\frac{1}{z_1 \bar{z}_2}$.

OR

- Represent the complex number $z = 1 + i$ in polar form.
23. Obtain all pairs of consecutive odd natural numbers such that in each pair both are more than 50 and their sum is less than 120.
24. A line cuts off equal intercepts on the coordinate axes, then find the angle made by the line with the positive x -axis.
25. Show that the points $A(3, -1, 2)$, $B(1, 2, -4)$, $C(-1, 1, 2)$ and $D(1, -2, 8)$ form a parallelogram.

PART - C

III Answer any TEN questions:

10 × 3 = 30

26. Out of a group of 200 students (who know at least one language), 100 students know English, 80 students know Kannada, 70 students know Hindi. If 40 students know all the three languages, find the number of students who know exactly two languages.
27. Let $R : z \rightarrow z$ be a relation defined by $R = \{(a, b) \mid a, b \in z, a - b \in z\}$. Show that
- i) $\forall a \in z, (a, a) \in R$
 - ii) $(a, b) \in R \Rightarrow (b, a) \in R$
 - iii) $(a, b) \in R, (b, c) \in R \Rightarrow (a, c) \in R$

28. f and g are two real functions and $f(x) = \begin{cases} -x & , x < e \\ x & , x \geq e \end{cases}$

$$g(x) = \begin{cases} -1 & , x < 0 \\ 0 & , x = 0 \\ 1 & , x > 0 \end{cases} . \text{ Find } (f + g)(x), (fg)(x) \text{ and the range of } (fg)(x).$$

29. Show that $\sqrt{3} \operatorname{cosec} 20^\circ - \sec 20^\circ = 4$.

OR

Prove that $(\cos x + \cos y)^2 + (\sin x - \sin y)^2 = 4 \cos^2 \left(\frac{x+y}{2} \right)$.

30. Convert $\frac{i-1}{\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}}$ into polar form.

31. Solve the following system of inequalities graphically:

$$5x + 4y \leq 0, \quad x \geq 2, \quad y \geq 3.$$

32. If $x + iy = \frac{2+i}{2-i}$ prove that $x^2 + y^2 = 1$.

33. Find the image of $(2, 3)$ on the line $3x + 5y = 4$.

OR

Reduce the equation $\sqrt{3}x + y - 8 = 0$ to normal form. Find the values of P and ω .

34. An amount Rs.500/- is deposited in a bank. If the bank pays an interest at the rate of 10% compounded annually, how much will be the amount in 10 years?

35. Find the derivative of the function 'cos x ' w.r.t ' x ' from first principle.

36. A parabola with vertex at the origin has its focus at the centre of $x^2 + y^2 - 10x + 9 = 0$. Find its directrix and latus rectum.

37. If $f(x) = \frac{x^2 - 4}{x\sqrt{x} - 2\sqrt{2}}$ find $\lim_{x \rightarrow 2} f(x)$ and see whether it equals $f(2)$.

38. Verify by the method of contradiction that $\sqrt{2}$ is irrational.

OR

Two students Anil and Sunil appear in an examination. The probability that Anil will qualify in the examination is 0.05 and that Sunil will qualify is 0.10. The probability that both will qualify the examination is 0.02. Find the probability that Anil and Sunil will not qualify in the examination.

PART – D

IV Answer any FOUR questions:

4 × 5 = 20

39. Prove that $\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1$ (θ being in radians) and hence show that

$$\lim_{\theta \rightarrow 0} \frac{\tan \theta}{\theta} = 1.$$

40. Prove by mathematical induction that $1^3 + 2^3 + 3^3 + \dots + n^3 = \frac{n^2(n+1)^2}{4}$.

41. A group consists of 7 boys and 5 girls. Find the number of ways in which a team of 5 members can be selected so as to have atleast one boy and one girl.

42. For all reals a, b and positive integer 'n' prove that,

$$(a + b)^n = {}^n C_0 a^n + {}^n C_1 a^{n-1} b + {}^n C_2 a^{n-2} b^2 + \dots + {}^n C_{n-1} a b^{n-1} + {}^n C_n b^n \text{ and}$$

$$\text{hence prove that } C_0 + C_1 + C_2 + \dots + C_n = 2^n.$$

43. Derive an expression for the coordinates of a point that divides the line joining the points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ internally in the ratio $m : n$. Hence, find the coordinates of the midpoint of AB where $A \equiv (1, 2, 3)$ and $B \equiv (5, 6, 7)$.

44. Define ellipse. Derive its equation in the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ($a > b$).

45. Find the mean deviation about the mean for the following data

Marks obtained	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Number of students	2	3	8	14	8	3	2

PART– E

V Answer any ONE question:

1 × 10 = 10

46. a) Derive a formula for the angle between two lines with slopes m_1 and m_2 .

Hence find the slopes of the lines which make an angle $\frac{\pi}{4}$ with the line $x - 2y + 5 = 0$.

b) Find the sum to n terms of the series $1^2 + (1^2 + 2^2) + (1^2 + 2^2 + 3^2) + \dots$

47. a) Prove geometrically that $\cos(A + B) = \cos A \cos B - \sin A \sin B$ and hence show that $\cos 2A = \cos^2 A - \sin^2 A$.

b) If $f(x) = \begin{cases} a + bx & , \quad x < 1 \\ 4 & , \quad x = 1 \\ b - ax & , \quad x > 1 \end{cases}$ and if $\lim_{x \rightarrow 1} f(x) = f(1)$ what are the possible values of 'a' and 'b'?
