Topic: Matrices

Question bank with solutions

One mark question (V S A)

1. Define matrix

2. Define a diagonal matrix

3. Define scalar matrix

4. Define symmetric matrix

5. Define skew-symmetric matrix

6.In a matrix $\begin{bmatrix} 2 & 5 & 19 & -17 \\ 35 & -2 & \frac{5}{2} & 12 \\ \sqrt{3} & 1 & -5 & 17 \end{bmatrix}$

find 1) order of the matrix

2) Write the elements of a_{13} , a_{21} , a_{33} , a_{24} , a_{23}

7. If a matrix 8 elements what is the possible order it can have ?

8. If a matrix 18 elements what is the possible order it can have?

9. construct 2×2 matrix $[a_{ij}]$ whose elements are given by

1) $a_{ij} = (i + j)^{2}$ 2) $a_{ij} = \frac{(i+j)^{2}}{2}$

10. construct the 2×3 matrix whose elements are given by $a_{ij} = |i - j|$

11. Construct the 3×2 matrix whose elements are given by $a_{ij} = \frac{i}{j}$

12. Find x, y, z if $\begin{bmatrix} 4 & 3 \\ x & 5 \end{bmatrix} = \begin{bmatrix} y & z \\ 1 & 5 \end{bmatrix}$

13. Find x, y, z if $\begin{bmatrix} x+y & 2 \\ 5+z & xy \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$

14. Find the matrix x such that 2A + B + X =0 where A = $\begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix}$ and B = $\begin{bmatrix} 3 & -2 \\ 1 & 5 \end{bmatrix}$

15. If A = $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$ B = $\begin{bmatrix} 3 & -1 & 3 \\ -1 & 0 & 2 \end{bmatrix}$ Find 2A – B

16. Find X if Y = $\begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$ and 2X+Y = $\begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}$

17. Find X If X+Y = $\begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$ and X-Y = $\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

18. Simplify $\cos\theta \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} + \sin\theta \begin{bmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{bmatrix}$

19. Find X If $2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$

20. If A = $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ Find A + A^1

21. A = $\begin{bmatrix} 1 & -2 & 3 \\ 0 & 1 & 4 \end{bmatrix}$ and B = $\begin{bmatrix} 0 & 2 & 5 \\ 6 & -3 & 1 \end{bmatrix}$ Find 3A + 2B

22. if A = $\begin{bmatrix} \sin\theta & \cos\theta \\ -\cos\theta & \sin\theta \end{bmatrix}$ Verify A A^1 = I

23. if B = $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ verify B B^1= I

24. If A = $\begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$ B = $\begin{bmatrix} 1 & 5 & 7 \end{bmatrix}$ Find AB

25. Compute 1) $\begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$

2) $\begin{bmatrix} 3 & -1 & 3 \\ -1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 2 & -3 \\ 1 & 0 \\ 3 & 1 \end{bmatrix}$

26. Find X and Y $\begin{bmatrix} 2x+y & 3y \\ 6 & 4 \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 6 & 4 \end{bmatrix}$

27. What is the number of possible square matrix order 3 with each entries 0 or 1

28. Find X and Y if $\begin{bmatrix} 5-x & 2y-8 \\ 0 & 3 \end{bmatrix}$ is a scalar matrix

29. Find X $\begin{bmatrix} 4 & x+2 \\ 2x-3 & x+1 \end{bmatrix}$ is a symmetric matrix

II. Two mark and Three marks questions (SA)

1.Radha , fauzia, simran are the student of 12^{th} class Radha has 15 note book and 6 pens , Fauzia has 10 books 2 pens and Simran has 13 books and 5 pens express this in to matrix forms.

2. Construct 3× 2 matrix whose elements are given by $a_{ij} = \frac{1}{2}\ |i - 3j|$

3. Find X,Y,Z from the equation $\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix}$ = $\begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$

4. Find a,b,c, d From the equation $\begin{bmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{bmatrix}$ = = $\begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$

5. If A = $\begin{bmatrix} 8 & 0 \\ 4 & -2 \\ 3 & 6 \end{bmatrix}$ B = $\begin{bmatrix} 2 & -2 \\ 4 & 2 \\ -5 & 1 \end{bmatrix}$ Find X such that 2A + 3X = 5B

6. Find X and Y 2$\begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix}$ + $\begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix}$ = $\begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$

7. Find X and Y if x $\begin{bmatrix} 2 \\ 3 \end{bmatrix} + y \begin{bmatrix} -1 \\ 1 \end{bmatrix}$= $\begin{bmatrix} 10 \\ 5 \end{bmatrix}$

8. Given 3 $\begin{bmatrix} x & y \\ z & w \end{bmatrix}$ = $\begin{bmatrix} x & 6 \\ -1 & 2w \end{bmatrix}$ + $\begin{bmatrix} 4 & x+y \\ z+w & 3 \end{bmatrix}$ Fine the values of X,Y,Z and W

9. If A_X = $\begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$ and A_Y = $\begin{bmatrix} \cos y & \sin y \\ -\sin y & \cos y \end{bmatrix}$ Show that $A_X A_Y$ = A_{X+Y}

10. If A = $\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$ and I = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ Find K If A^2 = KA – 2I

11. If A = $\begin{bmatrix} 3 & \sqrt{3} & 2 \\ 4 & 2 & 0 \end{bmatrix}$ and B =$\begin{bmatrix} 2 & -1 & 2 \\ 1 & 2 & 4 \end{bmatrix}$ verify $(A + B)^1 = A^1 + B^1$

12. For any matrix A with real number entries , A+ A^1 is symmetric matrix and A – A^1 Skew-symmetric matrix

13. For any matrix A = $\begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix}$ verify that A+ A^1 is symmetric matrix

14. For any matrix A = $\begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix}$ verify that A – A^1 Skew-symmetric matrix

15. If A and B be the invertible matrices of same order then $(AB)^{-1} = B^{-1}A^{-1}$

16. By using elementary operation Find the inverse of the matrix $\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$

17. By using elementary operation Find the inverse of the matrix $\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}$

18. By using elementary operation Find the inverse of the matrix $\begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}$

19. Find P^{-1} if it exists and P = $\begin{bmatrix} 10 & -2 \\ -5 & 1 \end{bmatrix}$

20. If A = $\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ Show that A^2 -5A +7I = 0

21. If A = $\begin{bmatrix} 2 & 3 \\ 0 & -4 \end{bmatrix}$ and B = $\begin{bmatrix} 1 & 5 \\ 2 & 0 \end{bmatrix}$ Show that $(AB)^1 = B^1A^1$

III. Five mark questions (LA)

1.If A = $\begin{bmatrix} 1 & 1 & -1 \\ 2 & 0 & 3 \\ 3 & -1 & 2 \end{bmatrix}$ B = $\begin{bmatrix} 1 & 3 \\ 0 & 2 \\ -1 & 4 \end{bmatrix}$ and C =$\begin{bmatrix} 1 & 2 & 3 & -4 \\ 2 & 0 & -2 & 1 \end{bmatrix}$

Find A B , BC and show that (AB)C = A(BC)

2. If A = $\begin{bmatrix} 0 & 6 & 7 \\ -6 & 0 & 8 \\ 7 & -8 & 0 \end{bmatrix}$ B = $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix}$ C = $\begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix}$ calculate AC, BC and (A+B) C

Deduce that (A+B) C = AC + BC

3. If A = $\begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$ Show that A^3 – 23A - 40 I = 0

4. If A = $\begin{bmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix}$ B = $\begin{bmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{bmatrix}$ and C = $\begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & -2 & 3 \end{bmatrix}$

verify A+ (B-C) = (A+B) –C

5. If A = $\begin{bmatrix} \frac{2}{3} & 1 & \frac{5}{3} \\ \frac{1}{3} & \frac{2}{3} & \frac{4}{3} \\ \frac{7}{3} & 2 & \frac{2}{3} \end{bmatrix}$ and B = $\begin{bmatrix} \frac{2}{5} & \frac{3}{5} & 1 \\ \frac{1}{5} & \frac{2}{5} & \frac{4}{5} \\ \frac{7}{5} & \frac{6}{5} & \frac{2}{5} \end{bmatrix}$ find 3A – 5B

6. If A = $\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$ find A^2 – 5 A + 6 I ?

7. If A = $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$ prove that $A^3 - 6A^2 + 7A + 2I = 0$

8. Express the matrix B = $\begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ Find the sum of symmetric and skew-symmetric matrix

9. Express the matrix B = $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ Find the sum of symmetric and skew-symmetric matrix

10. If A = $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ B = $\begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix}$ C = $\begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}$ calculate AB , BC, A(B+C)
Verify that AB + AC = A(B+C)

11. If F(x) = $\begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$ show that F(x) F(y) = F(x+y)

12. If A = $\begin{bmatrix} -2 \\ 4 \\ 5 \end{bmatrix}$ and B = $\begin{bmatrix} 1 & 3 & -6 \end{bmatrix}$ verify $(AB)^1 = B^1A^1$

13. If A = $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ Prove that A^n = $\begin{bmatrix} \cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta \end{bmatrix}$

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Solutions

One mark questions (VSA)

1. The numbers arranged in rectangular array of rows and columns by the brockets is called matrix
2. A square matrix is said to be diagonal matrix if all non diagonals elements are zeros
3. A diagonal matrix is said to be scalar marics if it's diagonal elements are equal
4. If a square matrix A = $[a_{ij}]_{m\times m}$is said to be symmetric if and only if A^1 = A
5. If a square matrix A = $[a_{ij}]_{m\times m}$is said to be skew-symmetric if and only if A^1 = -A
6. 1) order of the matrix is 3×4 2) 19, -2, -5 , 12, $\frac{5}{2}$
7. Possible orders are (1,8) (8,1) (2,4) (4,2) is 1X8 , 8X 1 , 2X 4 , 4X 2
8. Possible orders are (1,18) (18,1) (3,6) (6,3) (2,9) (9,2) is 1 X18 , 18X1 , 3X6 , 6X3 , 2X9, 9X2 ,
9. 1) $[a_{ij}] = \begin{bmatrix} 4 & 9 \\ 9 & 16 \end{bmatrix}$ 2) $[a_{ij}] = \begin{bmatrix} 2 & \frac{9}{2} \\ \frac{9}{2} & 8 \end{bmatrix}$
10. $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 2 \end{bmatrix}$
11. $\begin{bmatrix} 1 & \frac{1}{2} \\ 2 & 1 \\ 3 & \frac{3}{2} \end{bmatrix}$
12. X= 1 Y = 4 Z= 3
13. X = 2 Y = 4 Z = 0
14. X = -2A - B = $\begin{bmatrix} 2 & -4 \\ -6 & -8 \end{bmatrix} - \begin{bmatrix} 3 & -2 \\ 1 & 5 \end{bmatrix} = \begin{bmatrix} -1 & -2 \\ -7 & -13 \end{bmatrix}$
15. 2A –B = $\begin{bmatrix} -1 & 5 & 3 \\ 5 & 6 & 0 \end{bmatrix}$
16. By solving X = $\begin{bmatrix} -1 & -1 \\ -2 & -1 \end{bmatrix}$
17. By solving above matrix X = $\begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix}$ and Y = $\begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$
18. By multiplying we get the answer $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ = I
19. 2+Y = 5 implies Y = 3 and 2x+2 = 8 implies x =3
20. **20.** A + A^1 = $\begin{bmatrix} 2 & 5 \\ 5 & 8 \end{bmatrix}$

21. 3 A +2 B = $\begin{bmatrix} 3 & -2 & 19 \\ 12 & -3 & 14 \end{bmatrix}$

22. A A[1] = $\begin{bmatrix} \sin\theta & \cos\theta \\ -\cos\theta & \sin\theta \end{bmatrix} \begin{bmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{bmatrix}$ = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ = I after multiplying

23. BB [1] = $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$= I after multiplying

24. (AB)[1] = $\begin{bmatrix} 0 & 1 & 2 \\ 0 & 5 & 10 \\ 0 & 7 & 14 \end{bmatrix}$

25. 1) $\begin{bmatrix} -3 & -4 & 1 \\ 8 & 13 & 3 \end{bmatrix}$ 2) $\begin{bmatrix} 14 & -6 \\ 4 & 5 \end{bmatrix}$ after multiplying

26. 3 $|A| =$ K $|A|$ implies K = 3

27. Y = 0, X = 3 by solving

28. The square matrix of order 3X3 = 9 and 2 entries

Then possible entries is 2^9 = 512

29. $\begin{bmatrix} 5-x & 2y-8 \\ 0 & 3 \end{bmatrix}$= $\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$ then X = 2 Y = 4

30. $\begin{bmatrix} 4 & x+2 \\ 2x-3 & x+1 \end{bmatrix}$ = $\begin{bmatrix} 4 & 2x-3 \\ x+2 & x+1 \end{bmatrix}$ implies X = 5

Solutions : Two mark and Three marks questions (SA)

1.

	books	pens
Radha :	15	6
Fauzia :	10	2
Simran:	13	5

this can be expressed as $\begin{bmatrix} 15 & 6 \\ 10 & 2 \\ 13 & 5 \end{bmatrix}$ or $\begin{bmatrix} 15 & 10 & 13 \\ 6 & 2 & 5 \end{bmatrix}$

2. a_{ij} = $\begin{bmatrix} 1 & \frac{5}{2} \\ \frac{1}{2} & 2 \\ 0 & \frac{3}{2} \end{bmatrix}$

3. X+ Y + Z = 9 X +Z = 5 Y + Z = 7

7 + Z = 9 X + 2 = 5 Y + 2 = 7

Z= 2 X = 3 Y = 5

4. By solving equality a =1 , b= 2, c =3 and d = 4

5. X = $\begin{bmatrix} -2 & -\frac{10}{3} \\ 4 & \frac{14}{3} \\ -\frac{31}{3} & \frac{-7}{3} \end{bmatrix}$

6. compare two matrices X = 2, Y = 9

7. by solving we get X = 3 , Y = -4

8. by solving and compare we get X = 2 , Y = 4, Z = 1, w = 3

9. $A_X\ A_Y = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix} \begin{bmatrix} \cos y & \sin y \\ -\sin y & \cos y \end{bmatrix} = \begin{bmatrix} \cos(x+y) & \sin(x+y) \\ -\sin(x+y) & \cos(x+y) \end{bmatrix} = A_{X+Y}$

10. $A^2 = KA - 2I$

$\begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = \begin{bmatrix} 3k-2 & -2k \\ 4k & -2k-2 \end{bmatrix}$

Then 4K = 4

K = 1

11. $(A+B)^1 = \begin{bmatrix} 5 & 5 \\ \sqrt{3}-1 & 4 \\ 4 & 4 \end{bmatrix}$ and $A^1 + B^1 = \begin{bmatrix} 5 & 5 \\ \sqrt{3}-1 & 4 \\ 4 & 4 \end{bmatrix}$

Hence $(A+B)^1 = A^1 + B^1$

12. $B = A + A^1$, $B^1 = (A + A^1)^1 = A^1+A = B$ ∴ $B = A + A^1$ is symmetric

$C = A - A^1$, $C^1 = (A - A^1)^1 = A^1 - A = -(A - A^1) = -C$ ∴ $C = A - A^1$ is skew- symmetric

13. $Z = A + A^1 = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix} + \begin{bmatrix} 1 & 6 \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} 2 & 11 \\ 11 & 14 \end{bmatrix} = Z^1$ ∴ $Z = Z^1 = A + A^1$ is symmetric

14. $Z^1 = (A - A^1)^1 = \left(\begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix} - \begin{bmatrix} 1 & 6 \\ 5 & 7 \end{bmatrix}\right)^1 = \left(\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\right)^1 = -Z$ ∴ $Z^1 = -Z$, $A - A^1$ skew- symmetric

15. $(AB)(AB)^{-1} = I$

$A^{-1}(AB)(AB)^{-1} = A^{-1}I$	$IA = A$
$B(AB)^{-1} = A^{-1}$	$IA^{-1} = A^{-1}$
$B^{-1}B(AB)^{-1} = B^{-1}A^{-1}$	$AA^{-1} = I$
$(AB)^{-1} = B^{-1}A^{-1}$	BB^{-1} I

16. $A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$

A = IA

$\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A \quad R_2 = R_2 - 2R_1$

$\begin{bmatrix} 1 & 2 \\ 0 & -5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} A \quad R_2 = -\frac{1}{5} R_2$

$\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{2}{5} & -\frac{1}{5} \end{bmatrix} A \quad R_1 = R_1 - 2R_2$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{5} & \frac{2}{5} \\ \frac{2}{5} & -\frac{1}{5} \end{bmatrix} A$

$\therefore A^{-1} = \begin{bmatrix} \frac{1}{5} & \frac{2}{5} \\ \frac{2}{5} & -\frac{1}{5} \end{bmatrix}$

17. By above process $\therefore A^{-1} = \begin{bmatrix} 7 & -3 \\ -2 & 1 \end{bmatrix}$

18. By above prose's $\therefore A^{-1} = \begin{bmatrix} \frac{1}{5} & \frac{2}{5} \\ \frac{-2}{5} & \frac{1}{5} \end{bmatrix}$

19. $P = \begin{bmatrix} 10 & -2 \\ -5 & 1 \end{bmatrix}$

P = IP

$\begin{bmatrix} 10 & -2 \\ -5 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} P$

By elementary operation

$\begin{bmatrix} 1 & -\frac{1}{5} \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{10} & 0 \\ \frac{1}{2} & 1 \end{bmatrix} p$

p^{-1} does not exists

20. $A^2 - 5A + 7I = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$

21. By mathematical induction we get the solution

22. If $A=A^1$, $B = B^1$, $(AB)^1 = AB$

$(AB)^1 = B^1 A^1 = BA$ $\quad\therefore AB = BA \quad$ AB Is symmetric

23. A^2 = A A By product of two matrix get the solution

24. $(AB)^1 = \begin{bmatrix} 8 & -8 \\ 10 & 0 \end{bmatrix}$

$B^1A^1 = \begin{bmatrix} 8 & -8 \\ 10 & 0 \end{bmatrix}$

$\therefore (AB)^1 = B^1A^1$

25. By solving x = 2 , y = 4 , z = 3

Solutions : Five mark questions (LA)

1.AB = $\begin{bmatrix} 2 & 1 \\ -1 & 18 \\ 1 & 5 \end{bmatrix}$ (AB) C = $\begin{bmatrix} 4 & 4 & 4 & -7 \\ 35 & -2 & -39 & 22 \\ 31 & 2 & -27 & 11 \end{bmatrix}$

BC = $\begin{bmatrix} 7 & 2 & -3 & -1 \\ 4 & 0 & -4 & 2 \\ 7 & -2 & -11 & 8 \end{bmatrix}$ A(BC) = $\begin{bmatrix} 4 & 4 & 4 & -7 \\ 35 & -2 & -39 & 22 \\ 31 & 2 & -27 & 11 \end{bmatrix}$

Hence (AB) C = A(BC)

2. (A +B) C = $\begin{bmatrix} 10 \\ 20 \\ 28 \end{bmatrix}$ AC = $\begin{bmatrix} 9 \\ 12 \\ 30 \end{bmatrix}$ BC = $\begin{bmatrix} 1 \\ 8 \\ -2 \end{bmatrix}$ AC + BC = $\begin{bmatrix} 10 \\ 20 \\ 28 \end{bmatrix}$

Hence (A +B) C = AC + BC

3. A = $\begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$ A^2 = $\begin{bmatrix} 19 & 4 & 8 \\ 1 & 12 & 8 \\ 14 & 6 & 15 \end{bmatrix}$ A^3 = $\begin{bmatrix} 63 & 46 & 69 \\ 60 & -6 & 23 \\ 92 & 46 & 63 \end{bmatrix}$

LHS = A3 – 23A – 40 I = 0 By simplification

4. A + (B – C) = $\begin{bmatrix} 0 & 0 & -3 \\ 9 & -1 & 5 \\ 2 & 1 & 1 \end{bmatrix}$ and (A+B) –C = $\begin{bmatrix} 0 & 0 & -3 \\ 9 & -1 & 5 \\ 2 & 1 & 1 \end{bmatrix}$

Hence A + (B – C) = (A+B) –C

5. 3A -5B = $\begin{bmatrix} 2 & 3 & 5 \\ 1 & 2 & 4 \\ 7 & 6 & 2 \end{bmatrix}$ - $\begin{bmatrix} 2 & 3 & 5 \\ 1 & 2 & 4 \\ 7 & 6 & 2 \end{bmatrix}$ = $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ = 0

6. $A^2 - 5A + 6I$ = $\begin{bmatrix} 1 & -1 & -3 \\ -1 & -1 & -10 \\ -5 & 4 & 4 \end{bmatrix}$ by simplification

7. If A = $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$ by calculating A^2 , A^3 take LHS = RHS

8. B = $\begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ by theorem number 2

B = $\frac{1}{2}$ (B +B ^{1}) + $\frac{1}{2}$ (B -B ^{1}) hence they are equal

9\. B = $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ by theorem number 2

B = $\frac{1}{2}$ (B +B ^{1}) + $\frac{1}{2}$ (B -B ^{1}) hence they are equal

11\. If AB = $\begin{bmatrix} 4 & 6 \\ 5 & 3 \end{bmatrix}$ AC = $\begin{bmatrix} 5 & 7 \\ 4 & 5 \end{bmatrix}$ A(B+C) = $\begin{bmatrix} 9 & 13 \\ 9 & 8 \end{bmatrix}$ = AB + AC

12\. F(x).F(y) = $\begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos y & -\sin y & 0 \\ \sin y & \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$

= $\begin{bmatrix} \cos(x+y) & -\sin(x+y) & 0 \\ \sin(x+y) & \cos(x+y) & 0 \\ 0 & 0 & 1 \end{bmatrix}$ = F(x+y)

13\. LHS = $(AB)^1$ = $\begin{bmatrix} -2 & 4 & 5 \\ -6 & 12 & 15 \\ 12 & -24 & -30 \end{bmatrix}$ = B^1A^1 = RHS

14\. By mathematical induction we get the solution