INVERSE TRIGONOMETRIC FUNCTIONS

1 MARK QUESTIONS WITH SOLUTIONS

1. **Find the principal value of $sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$**

Solution let $sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = y, where\ \frac{-\pi}{2} < y < \frac{\pi}{2}$

$=> siny = \frac{1}{\sqrt{2}} => siny = sin\ \frac{\pi}{4} => y = \frac{\pi}{4}$

$=> the\ principal\ value\ of\ sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$

2. **Find the principal value of $cosec^{-1}(-\sqrt{2})$**

$=> let\ cosec^{-1}\left(-\sqrt{2}\right) = y, \quad where\ \frac{-\pi}{2} < y < \frac{\pi}{2}, y \neq 0$

$=> cosecy = -\sqrt{2}$

$=> cosecy = cosec\left(\frac{-\pi}{4}\right) \quad | \therefore cosec(-\theta) = -cosec\theta$

$=> y = \frac{-\pi}{4}$

$\therefore the\ principal\ value\ of\ cosec^{-1}\left(-\sqrt{2}\right) is -\left(\frac{\pi}{4}\right)$

3. Find the principal value of $sin^{-1}(sin\ \left(\frac{2\pi}{3}\right)$

Solution we have $sin^{-1}(sin\left(\frac{2\pi}{3}\right) \neq \frac{2\pi}{3}$ $\quad | -\frac{\pi}{2} < \frac{\pi}{3} < \frac{\pi}{2}$

$\therefore$ it does not lie b/w $\frac{-\pi}{2}$ and $\frac{-\pi}{2}$

$sin^{-1}(sin\left(\frac{2\pi}{3}\right) = sin^{-1}\left[sin\left(\pi - \frac{\pi}{3}\right)\right] = sin^{-1}\left[sin\left(\frac{\pi}{3}\right)\right] = \frac{\pi}{3}$

4. $Find\ the\ value\ of\ cos^{-1}\left(cos\ \frac{13\pi}{6}\right) \leq$

solution

we have $cos^{-1}\left(\cos\left(\frac{13\pi}{6}\right)\right) \neq \frac{13\pi}{6}$, it does not lie b/w 0 and π

$cos^{-1}\left(cos\ \left(\frac{13\pi}{6}\right)\right) = cos^{-1}\left(cos\ \left(2\pi + \frac{\pi}{6}\right)\right)$

$= cos^{-1}\left(cos\frac{\pi}{6}\right)$

$= \frac{\pi}{6}$

$\therefore the\ principal\ value\ of\ cos^{-1}\left(cos\ \frac{13\pi}{6}\right) is\ \frac{\pi}{6}$

5. $S.T. sin^{-1}\left(2x\sqrt{1 - x^{-2}}\right) = 2sin^{-1}x \quad , \frac{1}{\sqrt{2}} \leq x \leq \frac{1}{\sqrt{2}}$

$solution\ \ let\ sin^{-1}x = \theta => x = sin\ \theta$

$we\ have\ sin^{-1}\left(2. sin\theta\sqrt{1 - sin^2\theta}\right)$

$=> sin^{-1}(2sin\theta. cot\theta)$

$=> sin^{-1}(\sin 2\ \theta) = 2\theta$

$=> 2\ sin^{-1}x$

6. *Evaluate* $sin^{-1}(sin(-600^0)$

we have $sin^{-1}\left(sin(-600^0)\right) = sin^{-1}\left[-sin(600^0)\right]$

$= sin^{-1}\left[-sin\,(360^0 + 240^0)\right]$

$= sin^{-1}\left[-\left(+sin(180^0 + 60^0)\right)\right]$

$= sin^{-1}\left[-(-sin60^0)\right]$

$= sin^{-1} = sin(60^0) = 60^0$

7. *Find the value of* $cos^{-1}\frac{1}{2} + 2\ sin^{-1}\frac{1}{2}$

Solution we have $cos^{-1}\frac{1}{2} + 2\ sin^{-1}\frac{1}{2}$

$=> \frac{\pi}{3} + 2.\frac{\pi}{6} = \frac{\pi}{3} + \frac{\pi}{3} = \frac{2\pi}{3}$

8. *Find the value of* $cot\ (tan^{-1} a + cot^{-1} a)$

we have $\cot(tan^{-1} a + cot^{-1} a) = \cot\left(\frac{\pi}{2}\right) = 0$

2 MARKS QUESTIONS WITH SOLUTIONS

1. $P.T.\ sin^{-1} x + cos^{-1} x = \frac{\pi}{2}, \quad x \in [-1,1,]$

solution $le\ sin^{-1} x = \theta =. x = \sin\theta\ with\ \frac{-\pi}{2} \le \theta \le \frac{\pi}{2}$

$=> sin\theta = x \qquad and - 1 \le x \le 1$

$=> \cos\left(\frac{\pi}{2} - \theta\right) = x \qquad |..\sin\theta = \cos\left(\frac{\pi}{2} - \theta\right)$

$=> \frac{\pi}{2} - \theta = cos^{-1} x$

$=> \frac{\pi}{2} = sin^{-1} + cos^{-1}$

2. $P.T. 2\ tan^{-1} x = cos^{+1}\left(\frac{1-x^2}{1+x^2}\right), \quad x \ge 0$

$solution.\ let\ tan^{-1} x = \theta => x = tan\theta$

$consider\ cos\ 2\theta = \left(\frac{1-tan^2\theta}{1+tan^2\theta}\right)$

$=> 2\theta = cos^{-1}\left(\frac{1-tan^2\theta}{1-tan^2\theta}\right)$

$=> 2\ tan^{-1} x = cos^{+1}\left(\frac{1-x^2}{1+x^2}\right)$

3. $P.T. 3\ cos^{-1} x = cos^{-1}\ (4x^3 - 3x), \qquad x \in \left[\frac{1}{2}, 1\right]$

$solution\ let\ cos^{-1} x = \theta => x = cos\ \theta$

$consider\ 3\theta = (4\ cos^3\theta - 3\cos\theta)$

$=> 3\theta = cos^{-1}(4\ cos^3\theta - 3\cos\theta)$

$=> 3\ cos^{-1} x = cos^{+1}(4x^3 - 3x)$

4. $P.T.\ 2tan^{-1}\frac{1}{2} + tan^{-1}\frac{1}{7} = tan^{-1}\left(\frac{31}{17}\right)$

solution $\qquad LHS = 2\ tan^{-1}\frac{1}{2} + tan^{-1}\frac{1}{7}$

$= tan^{-1}\left(\frac{2.\frac{1}{2}}{1-\frac{1}{4}}\right) + tan^{-1}\frac{1}{7} \qquad | \therefore\ 2\ tan^{-1} x = tan^{-1}\left(\frac{2x}{1-x^2}\right)$

$= tan^{-1}\left(\frac{1}{\frac{3}{4}}\right) + tan^{-1}\left(\frac{1}{7}\right)$

$= tan^{-1} \left[\frac{\frac{4}{3}+\frac{1}{7}}{1-\frac{4}{3}\cdot\frac{1}{7}}\right]$ $\quad | \therefore \ tan^{-1} x + tan^{-1} y = tan^{-1}\left(\frac{x+y}{1+xy}\right)$

$= tan^{-1}\left[\frac{\frac{31}{21}}{1-\frac{4}{21}}\right] = tan^{-1} \frac{31}{21} \times \frac{21}{17} = tan^{-1}\left(\frac{31}{17}\right)$

5) *Find the value of* $tan\frac{1}{2}\left[sin^{-1} \frac{2x}{1+x^2} + cos^{-1} \frac{1-y^2}{1+y^2}\right]$, $\quad |x| < 1$

Y > 0

|xy| < 1

solution $\quad W.K.T.\ sin^{-1} \frac{2x}{1-x^2} = 2\ tan^{-1} x\ for\ |x| < 1$

$and\ cos^{-1}\left(\frac{1-y^2}{1+y^2}\right) = 2\ tan^{-1} y \quad for\ y > 0$

$we\ have \tan\frac{1}{2}\ [\ 2\ tan^{-1} x + 2\ tan^{-1} y\ ]$

$= tan\ [\ tan^{-1} x + tan^{-1} y]$

$= \tan\left[tan^{+1}\left(\frac{x+y}{1-xy}\right)\right] = \left(\frac{x+y}{1-xy}\right)$

6) $solve\ 2\ tan^{-1}(cosx) = tan^{-1}(2cosec\ x)$

$solution\ we\ have\ 2\ tan^{-1}(cosx) = tan^{-1}(2cosec\ x)$

$w.k.t.\ 2\ tan^{-1} a = tan^{-1}\left(\frac{1-a}{1+a}\right)$

$=> tan^{-1}\left(\frac{2cosex}{1-cos^2 x}\right) = tan^{-1}\ (2\ cosec\ x)$, $\quad |\ 2\ tan^{-1} x$

$=> \frac{2cosx}{1-cos2x} = 2cosec\ x \qquad tan^{-1}\left(\frac{2x}{1= x^2}\right) \quad => \frac{2cosx}{sin^2 x} = 2\frac{1}{sinx}$

$=> \frac{cosx}{sinx} = 1 => 1 = tanx$

$= x = tan^{-1}(1)$

$x = \frac{\pi}{4}$

7) $solve\ \ tan^{-1}\left(\frac{1-x}{1+x}\right) = \frac{1}{2}\ tan^{-1} x, x > 0$

$we\ have\ tan^{-1}\left(\frac{1-x}{1+x}\right) = \frac{1}{2}\ tan^{-1} x$

$=> 2\ (\ tan^{-1} - tan^{=1} x) = tan^{=1} x$

$=> 2\left(\frac{\pi}{4}\right) - 2\ tan^{+1} x = tan^{+1} x$

$=> \frac{\pi}{2} = 3\ tan^{-1} x => \frac{\pi}{2} = tan^{-1} x => x = tan^{-1}\ \frac{\pi}{6} = \frac{1}{\sqrt{3}}\ \therefore\ x = \frac{1}{\sqrt{3}}$

8) $Write\ the\ function\ in\ simplest\ form\ tan^{-1} \frac{\sqrt{1+x^2}-1}{x}$, $\qquad x \neq 0$

$solution\ we\ have\ tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$

$put\ x = \tan\theta => \theta = tan^{-1} x$

$\therefore\ tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right) = tan^{-1}\left(\frac{\sqrt{1+tan^2\theta}-1}{tan\theta}\right)$

$= tan^{-1}\left(\frac{sec\theta-1}{tan\theta}\right)$

$= tan^{-1}\left(\frac{\frac{1}{cos\theta}-1}{\frac{sin\theta}{cos\theta}}\right) = tan^{-1}\left(\frac{1-cos\theta}{sin\theta}\right) = tan^{-1}\left(\frac{2sin^2\frac{\theta}{2}}{2sin\frac{\theta}{2}\ cos\frac{\theta}{2}}\right)$

$$= tan^{-1}\left(tan\frac{\theta}{2}\right) = \frac{\theta}{2} = \frac{1}{2}\ tan^{-1} x.$$

9) $Find\ the\ value\ of\ \ tan^{-1}\left(\frac{x}{y}\right) - \ tan^{-1}\left(\frac{x-y}{x+y}\right)$

$solution\ we\ have\ \ tan^{-1}\left(\frac{x}{y}\right) - \ tan^{-1}\left(\frac{x-y}{x+y}\right)$

$= tan^{-1}\left(\frac{x}{y}\right) - \ tan^{-1}\left[\frac{x\left(1-\frac{y}{x}\right)}{x\left(1+\frac{y}{x}\right)}\right]$

$= tan^{-1}\frac{x}{y} - \left[tan^{-1}\left(\frac{1-\frac{y}{x}}{1+\frac{y}{x}}\right)\right]$

$= tan^{-1}\frac{x}{y} + tan^{-1}\frac{y}{x} - tan^{-1} 1 \qquad | \therefore tan^{-1}\frac{y}{x} = cot^{-1}\left(\frac{x}{y}\right)$

$= (tan^{-1}\frac{x}{y} + tan^{-1}\frac{y}{x} - tan^{-1} 1 \qquad cot^{-1} a + tan^{-1} a = \frac{\pi}{2}$

$= \left(tan^{-1}\frac{x}{y} + cot^{-1}\frac{x}{y}\right) - \frac{\pi}{4}$

$= \frac{\pi}{2} - \frac{\pi}{4} = \frac{2\pi-\pi}{4} = \frac{\pi}{4}$

10) $solve\ \ tan^{-1} 2x + \ tan^{-1} 3x = \frac{\pi}{4}$

$so\ we\ have\ tan^{-1} 2x + \ tan^{-1} 3x = \frac{\pi}{4}$

$=> tan^{-1}\left(\frac{2x+3x}{1-2x.3x}\right) = \frac{\pi}{4}$

$=> tan^{-1}\left(\frac{5x}{1-6x^2}\right) = tan\frac{\pi}{4} => \frac{5x}{1-6x^2} = 1$

$=> 1 - 6x^2 + 5x = 0$

$=> 6x^2 - 5x - 1 = 0 \ => 6x^2 + 6x - 1.x - 1 = 0$

$=> 6x(x+1) - 1(x+1) = 0$

$\therefore (6x-1)(x+1) = 0$

$\therefore 6x - 1 = 0 \ \ or\ x + 1 = 0$

$6x = 1$

$x = \frac{1}{6}\ is\ only\ solution$

$but\ x = -1\ does\ not\ satisfies$

$\therefore LHS\ of\ equation\ become - ve$

3 marks Question with solutions

1. $If tan^{-1}\left(\frac{x-1}{x+2}\right) + tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}\ find\ the\ value\ of\ x$

$solution \qquad consider\ the\ equation$

$tan^{-1}\left(\frac{x-1}{x-2}\right) + tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}$

$=> tan^{-1}\left[\frac{\frac{x-1}{x-2}+\frac{x+1}{x+2}}{1-\left(\frac{x-1}{x-2}\right)\left(\frac{x+1}{x+2}\right)}\right] = \frac{\pi}{4}$

$$=> tan^{-1}\left[\frac{\frac{x-1}{x-2}+\frac{x+1}{x+2}}{1-\left(\frac{x-1}{x-2}\right)\left(\frac{x+1}{x+2}\right)}\right] = \tan\frac{\pi}{4}$$

$$=> [\left(\frac{(x+2)(x+1)+(x-2)(x+1)}{(x^2-4)-(x^2-1)}\right] = 1$$

$$=> \frac{2x^2-4}{-3} = 1 \quad => \quad 2x^2 - 4 = -3$$

$$=> 2x^2 = -3+4$$

$$=> 2x^2 = 1$$

$$=> x^2 = \frac{1}{2}$$

$$=> x = \pm\frac{1}{\sqrt{2}}$$

2) $S.T\ sin^{-1}\frac{3}{5} - sin^{-1}\frac{8}{17} = cos^{-1}\frac{84}{85}$

$solution\ let\ sin^{-1}\frac{3}{5} = x => \frac{3}{5} = sinx,\ \ sin^{-1}\frac{8}{17} = y,\ \frac{8}{17} = siny$

$$=> cosx = \sqrt{1-sin^2 x} = \sqrt{\frac{1-9}{25}} = \sqrt{\frac{25-9}{25}} = \frac{4}{5}, cosx = \frac{4}{5}.$$

$$cosy = \sqrt{1-sin^2 y} = \sqrt{1-\frac{64}{289}} = \sqrt{\frac{689-64}{289}} = \frac{15}{17}, cosy = \frac{15}{17}$$

$we\ have\ \cos(x-y) = cosx\ cosy + sinx\ siny$

$$= \frac{4}{5}\cdot\frac{15}{17} + \frac{3}{5}\cdot\frac{8}{17}$$

$$= \frac{60}{85} + \frac{24}{85}$$

$$= \frac{84}{85}$$

$$x - y = cos^{-1}\left(\frac{84}{85}\right)$$

$$= sin^{-1}\frac{3}{5} - sin^{-1}\frac{8}{17} = cos^{-1}\left(\frac{84}{85}\right)$$