Topic :- Linear Programming

The running of any firm or of a factory involves many constraints like financial,space,resources,power etc. The objective of any business person would be to make the profit maximum (in the case of investment to make the cost a minimum) under all the constraints.The linear programming problem is the problem of optimising an objective function under a given set of constraints.When the objective function is profit function the optimisation is to maximise the profit.In the case of cost function the optimisation is to minimise the cost.

All the constaints of the problem are linear inequalities and the objective function is linear.Hence, it is called LPP.The following are a few illustrations of the LPP by Graphical method .

1. Maximise : P = $3x + 2y$.

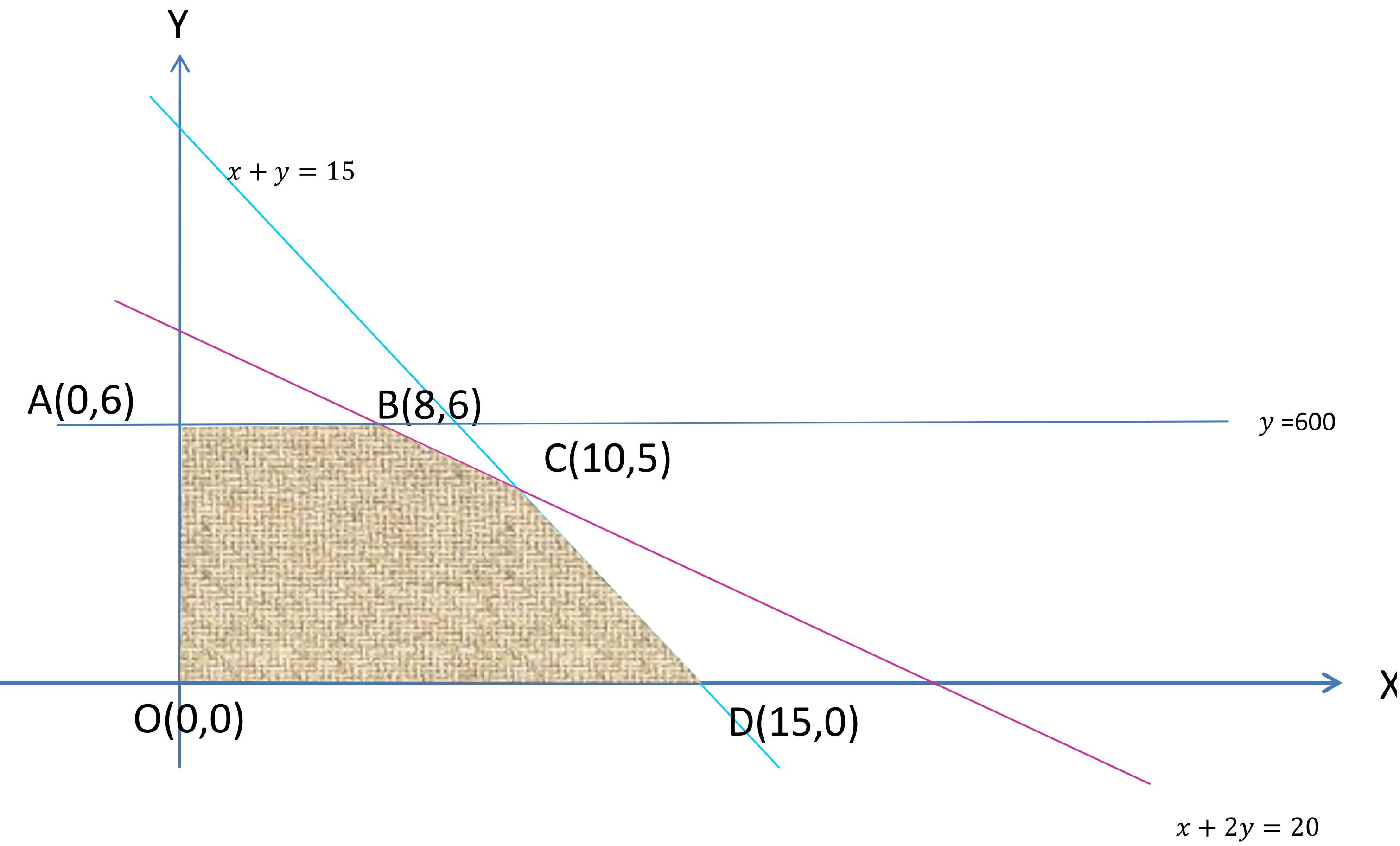
Subject to : $x + 2y \le 20$

$x + y \leq 15$

$y \le 6$

$x, y \geq 0.$

Now we draw the graphs of the equations $x + 2y = 20$, $x + y = 15$ and $y = 6$ and recognise the feasible region.



The shaded region OABCDO (bounded) represents the feasible region in which all the Constraints of the problem are satisfied. Now, we evaluate the objective function at these corners of the region.

Vertex	x	y	$P = 3x + 5y$
0	0	0	0
A	0	6	3X0+5X6=30
B	8	6	3X8+5X6=54
C	10	5	3X10+5X5=55
D	15	0	3X15+5X0=45

From the above table we observe that Maximum value of P is 55 and corresponds to

X = 10 and y = 5

2). Minimise: C $= 80\,x + 64y$

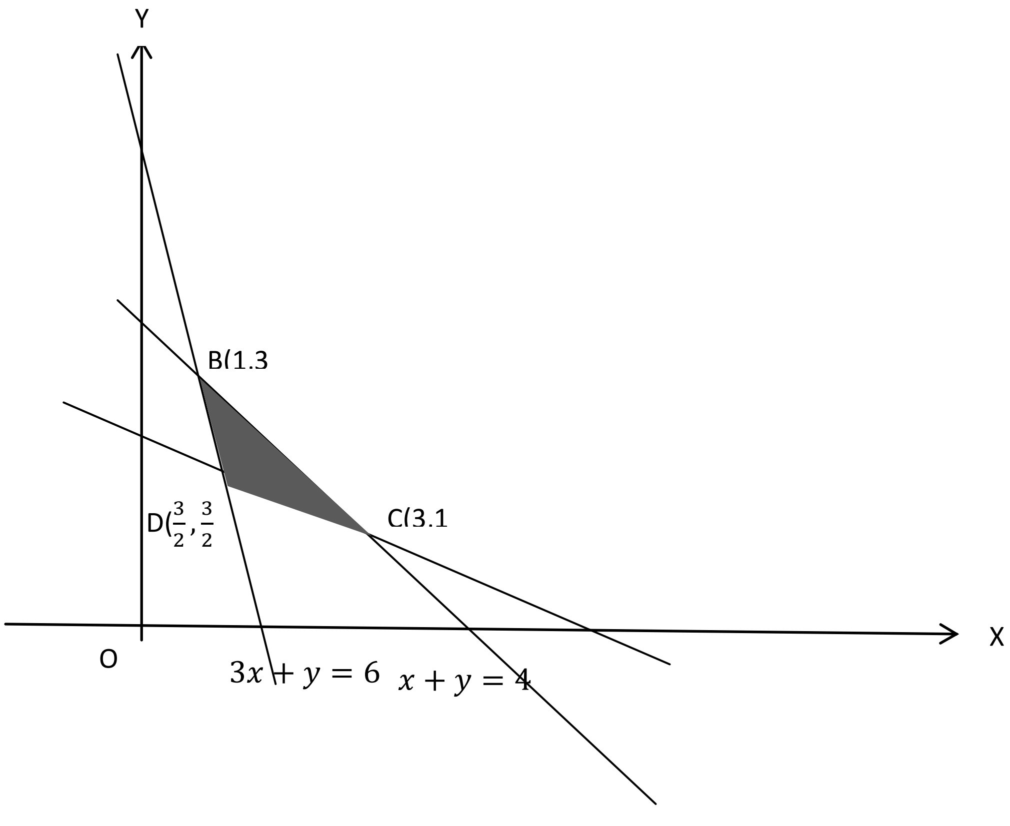
Subject to : $3x + y \geq 6$

$x + y \leq 4.$

$x + 3y \geq 6$

$x, y \geq 0\,.$

we draw the graphs of the linear equations $3x + y = 6\,, x + y = 4$ and $x + 3y = 6$



The shaded region BCD (triangular region ,bounded) is the feasible region.

In this region all the constraints of the problem are satisfied.Now we evaluate the objective function C = $80x + 64y$ at the verticies.

Vertex	x	y	C = $80x + 64y$
B	1	3	272
C	3	1	304
D	3/2	3/2	216

From the above table we observe that the minimum value of C is 216 and corresponds to $x = \frac{3}{2}$ and $y = \frac{3}{2}$.

Note:- The co-ordinates of the points B, C and D can be read from the graph. They may also be determined by solving the equations of the corresponding pair of lines

(3) Maximise : $Z = 5x + 2y$

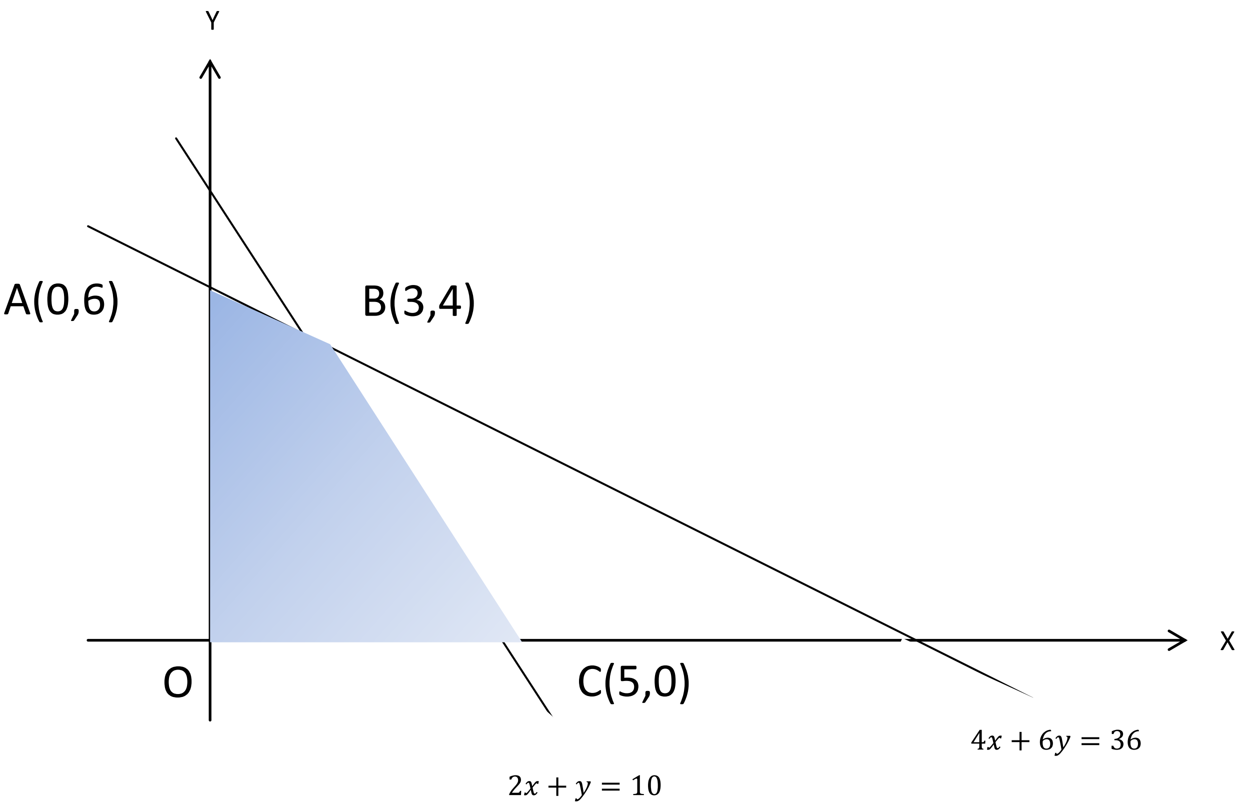
$x + 3y = 6$

Subject to : $4x + 6y \leq 36$

$2x + y \leq 10$

$x , y \geq 0.$

Now we draw the graphs of the linear equations and recognise the feasible region.



The shaded region OABCO is the feasible region. Now we evaluate $Z = 5x + 2y$ at the vertices.

Vertex	x	y	$Z = 5x + 2y$
0	0	0	0
A	0	6	12
B	3	4	23
C	5	0	25

From the table we find that the maximum value of Z = 25 and corresponds to $x = 5, y = 0$.
Note:The above working is based on the result that the optimal solution of a LPP if exists will occur at a corner point of the feasible region.

(4) Maximise $P = 2x + 3y$

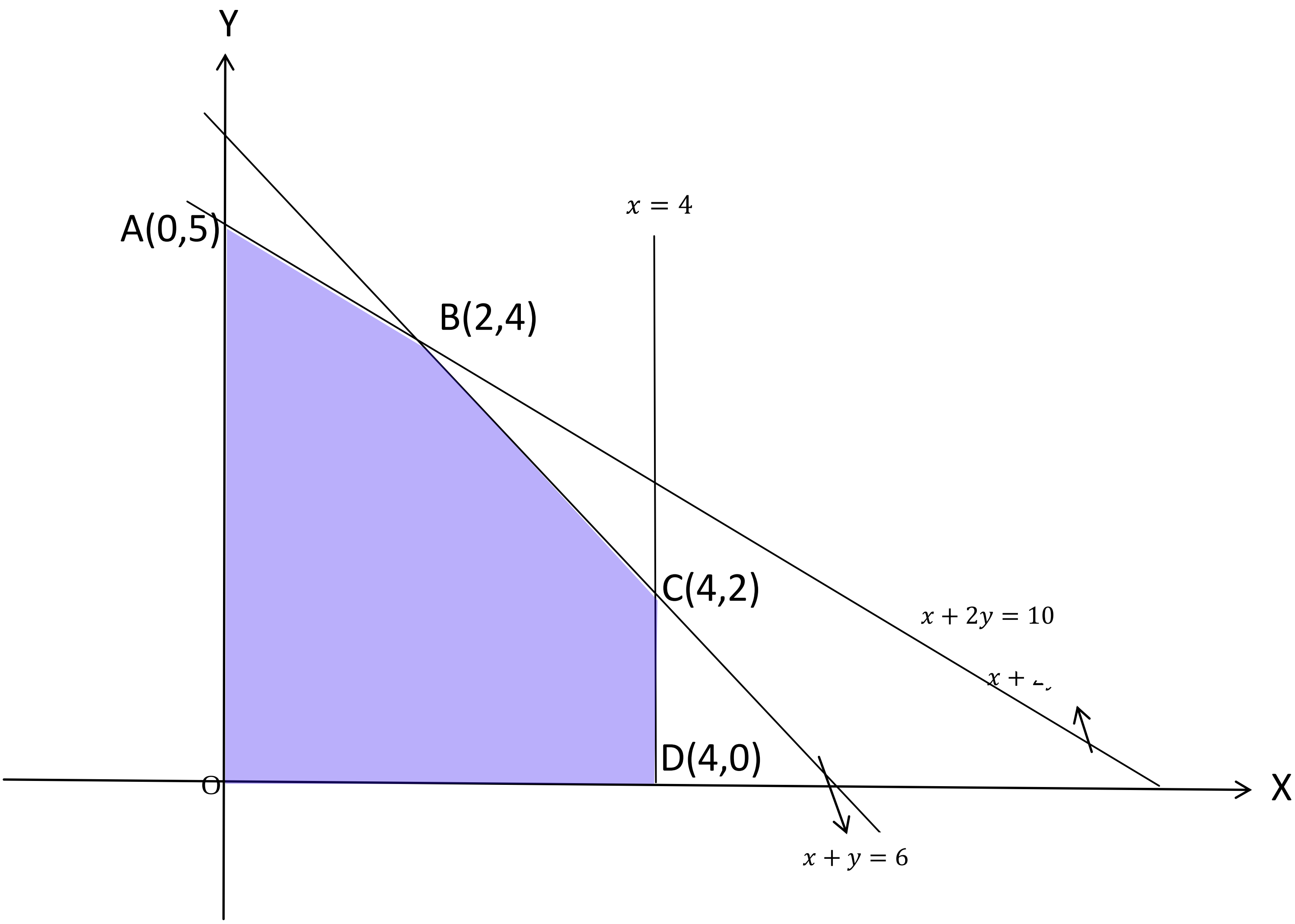
Subject to : $x + 2y \leq 10$

$x + y \leq 6$

$x \leq 4$

$x, y \geq 0$

We draw the graphs corresponding to the linear equations and shade the feasible region.



The shaded region OABCDO(bounded) is the feasible region.

Now we evaluate $P = 3\,x + 2\,y$ at the vertices

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Vertex	x	y	$p = 3x + 2y$
0	0	0	0
A	0	5	10
B	2	4	14
C	4	2	16
D	4	0	12

From the table we find that the maximum value of p = 16 and corresponds to $x = 4, y = 2$

(5). Determine the maximum and minimum values of

$Z = 4x + y$

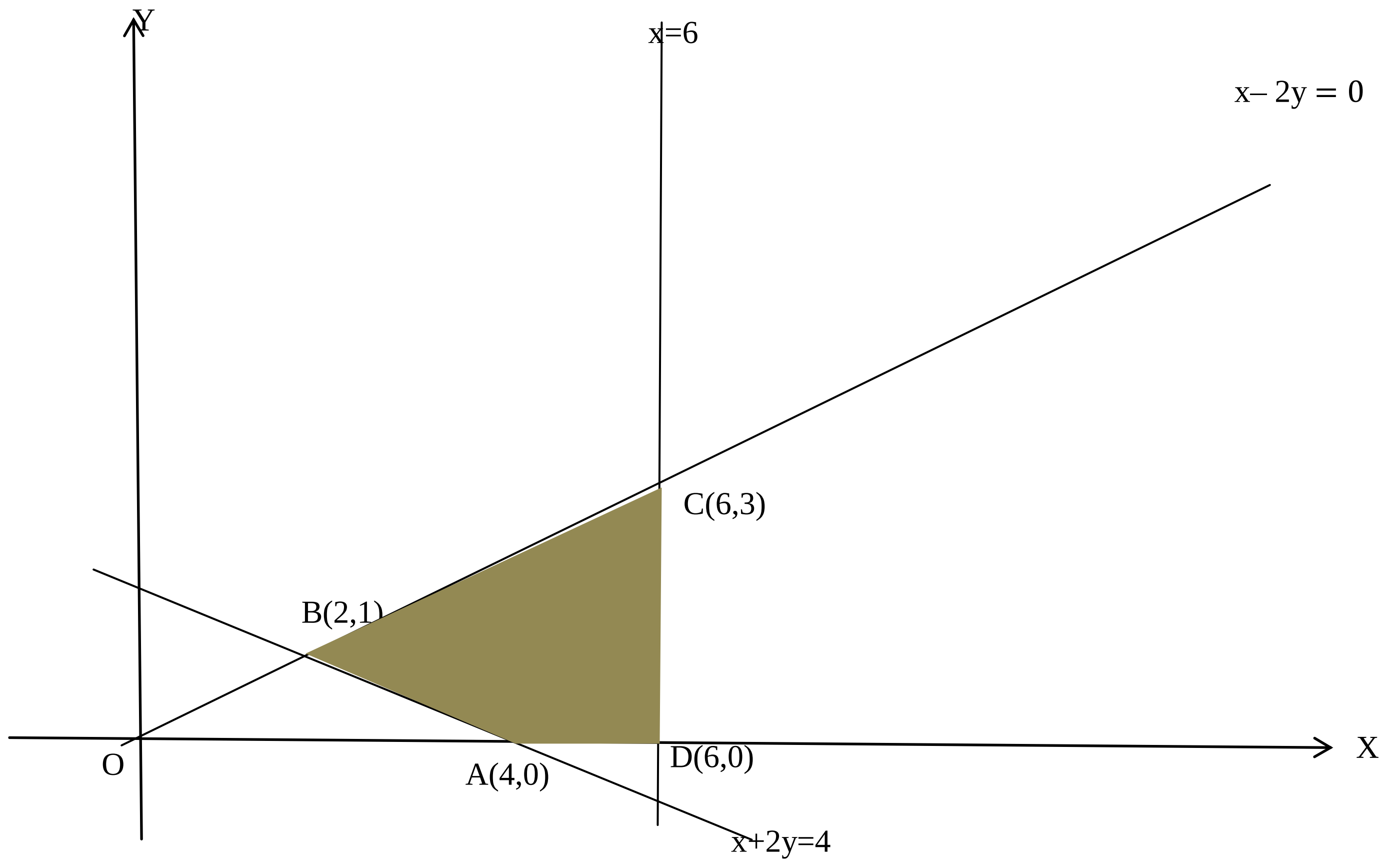
Subject to: $x + 2y \geq 4$

$x - 2y \geq 0$

$x \leq 6$

$x, y \geq 0$

solution: We draw the lines x+ 2y = 4 , x– 2y = 0 and x= 6.



The feasible region is bounded region with A(4, 0) , B (2 , 1) , C (6 , 3) and D(6 , 0)as vertices.
Now we find the values of Z = 4x + 2y at these vertices.

Vertex	X	Y	Z = 4x + 2 y
A	4	0	16
B	2	1	10
C	6	3	30
D	6	0	24

From the above table we find that the minimum value of Z is 10 and corresponds to x = 2 and y = 1. The maximum value of Z is 30 and corresponds to x = 6 and y =3.

APPLICATIONS PROBLEMS

1). A factory manufactures two types of screws, A and B by using two machines.The time required for the manufacture of one packet of each of the two types of the screws on the two machines,the total time available of each of the mechine and the profit obtained on the sale of each packet of the two types of the screws are given below.

Machine / Screws	Time required in minutes		Profit in Rs Per packet.
	I	II	
A	4	6	8
B	6	3	5
Time available in hours	4	4	--

Formulate this as a Lpp and determine the number of packets of each of the two types of screws to be manufactured so as to get maximum profit assuming that all the packets of the screws manufactured are sold.

Solution;-
Let x and y be the number of packets of the type A and type B screws to be produced.
Then,the total time the mechine I works is 4 x + 6y.
The total time the machine II works is 6x + 3y.
The total profit in Rs. is p = 8x + 5y.

Since, the two machines are available at most for 4 hours = 240 minutes, we have $4x + 6y \leq 240$. and $6x + 3y \leq 240$.

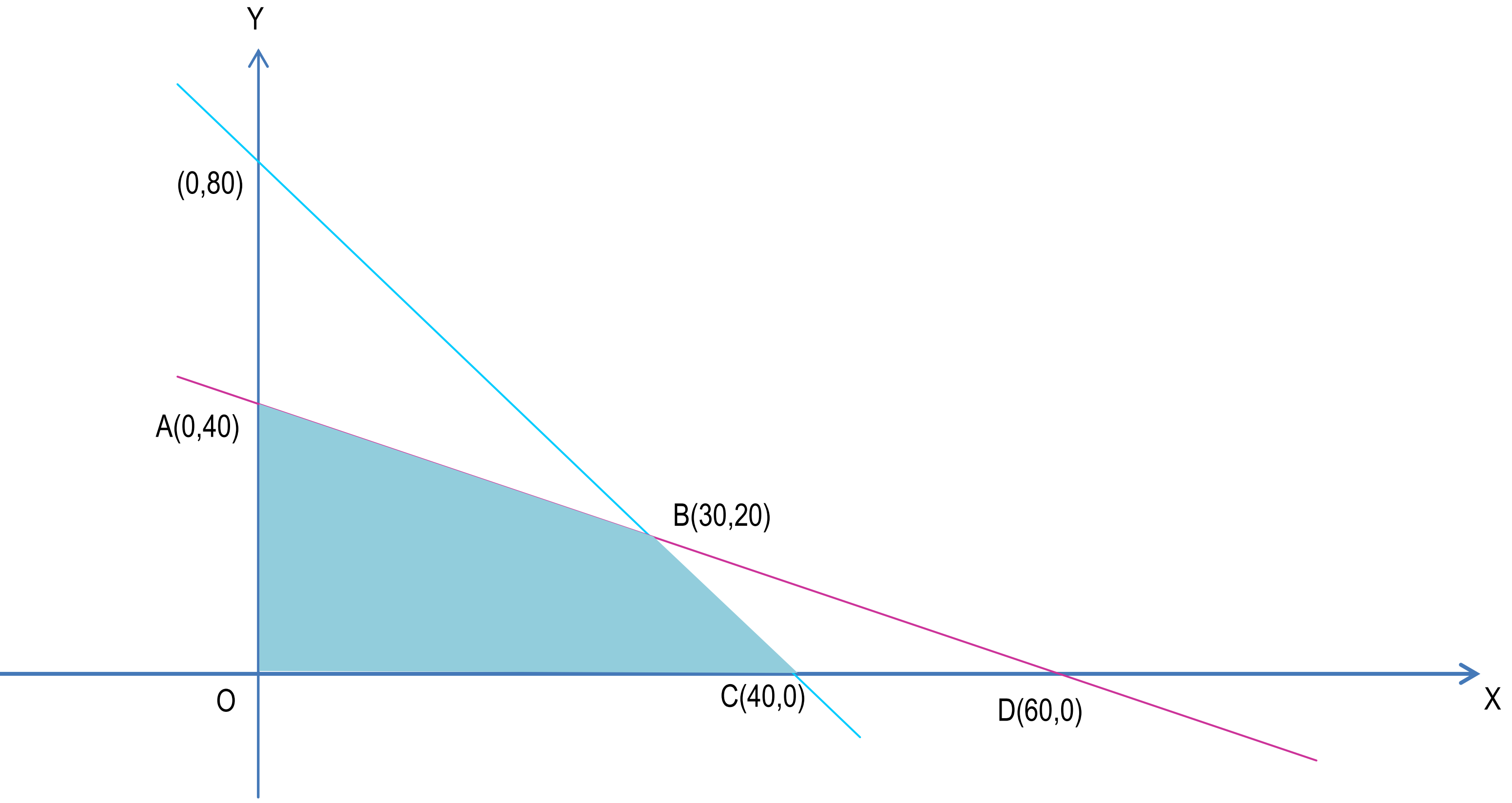
.`. the Lpp is to maximise $p = 8x + 5y$

Subject to:- $4x + 6y \leq 240$

$6x + 3y \leq 240$

$x, y \geq 0$

Now, we draw the graphs of the linear equations $4x + 6y = 240$ and $6x + 3y = 240$.



The feasible region is bounded region OABCO with O(0,0), A(0,40) , B (30,20) , and C(40 ,0) as vertices.

Now we find the values of $p = 8x + 5y$ at these vertices.

Vertex	X	Y	p = 8x + 5 y
O	0	0	0
A	0	40	200
B	30	20	340
C	40	0	320

From the table we find that the maximum value of p = 340 and corresponds to
$x = 30 \; and \; y = 20.$
`Hence, the maximum profit is Rs. 340 and corresponds to manufacture of 30 packets of A type.and 20 packets of B type screws.

2) The production of two types of suitcases requires processing and completion to be done on two machines A and B.The time required for processing and completion of each type of trunk on the two machines, the time available on each machine and profit on each type of the suit case is given below.

Suitcase / Machine	Time required in hours		Time available in hours
	Type I	Type II	
A	3	3	18
B	2	4	16
Profit in Rs Per suitcase	30	42	--

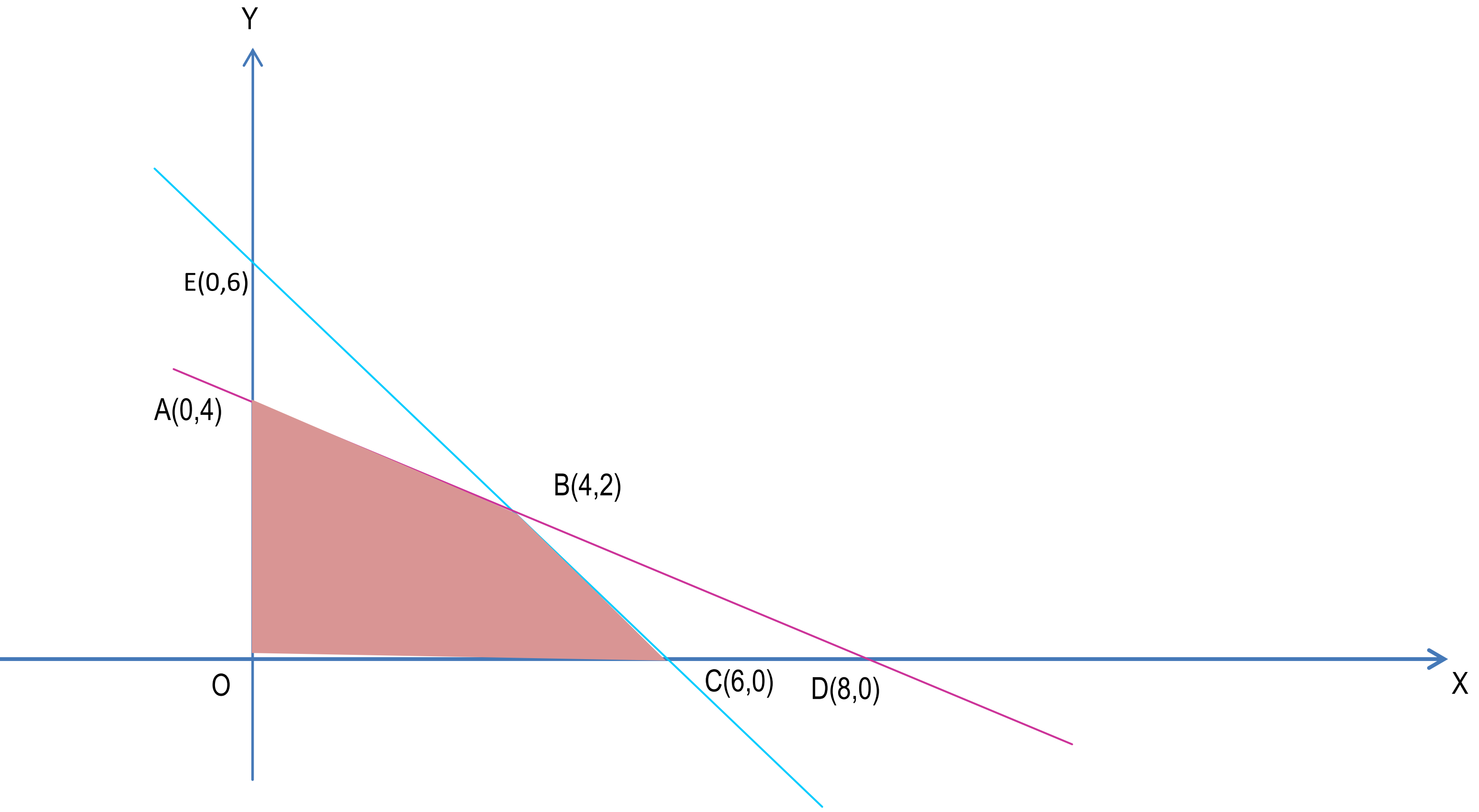
Determine the number of the two types of suitcases to be produced to get maximum profit ..

Solution;-
Let x and y be the number of type I and type II suitcases to be produced.
Then,the total time the mechine A works is 3 x + 3y.
The total time the machine B works is 2x + 4y.
The total profit in Rs. is p = 30x + 42y

Since, the two machines A and B are available at most for 18and 16 hours respectively, we have $3x + 3y \leq 18$. and $2x + 4y \leq 16$.

.`. the Lpp is to maximise $p = 30x + 42y$
Subject to:- $3x + 3y \leq 18$
$2x + 4y \leq 16$
$x, y \geq 0$

Now, we draw the graphs of the linear equations 3x + 3y = 18 and 2x +4y=16.



The feasible region is bounded region OABCO with O(0,0), A(0,4) , B (4,2) , and C(6 ,0) as vertices.All the condtions of the problem are satisfied within and on the boundary of this
Region.
Now we find the values of p = 30 x + 42 y at these vertices.

Vertex	x	Y	p = 30 x + 42 y
O	0	0	0
A	0	4	168
B	4	2	204
C	6	0	180

From the table we find that the maximum value of p = 204 and corresponds to $x = 4 \text{ and } y = 2.$
Hence, the maximum profit is Rs 204 and corresponds to manufacture of 4 suitcases of
type I.and 2 suitcases of type II..

3).At a cattle rearing , it is prescribed that the food for each animal contain at most 16 units of nutrient A, and at least 24 and 48 units of nutrients B and C respectively.Two types of fodders are available. The number of units of these nutrients contained per Kg by the two types of fodders and their cost per Kg is given below

Content of nutrient/kg	A	B	C	Cost per kg in Rs.
Fodder- 1	1	3	2	12
Fodder-2	1	1	6	15

By graphical method determine the number of kg of the two types of the fodder to be purchased so as to make the cost a minimum,yet meeting the requirements.

Solution:

Let, x and y kg of fodder-1and fodder-2 be purchased.

Then, the cost in Rs. Is $C = 12x + 15y$.

Total content s of nutrients A, B and C are $x + y$, $3x + y$ and $2x + 6y$ respectively.

Since the content of A is to at most 16 while it has to be at least 24 for B and 48 for C, the constraints are

$x + y \leq 16$, , $3x + y \geq 24$, and $2x + 6y \geq 48$.

$\therefore$.The Lpp is To minimise; $C = 12x + 15y$

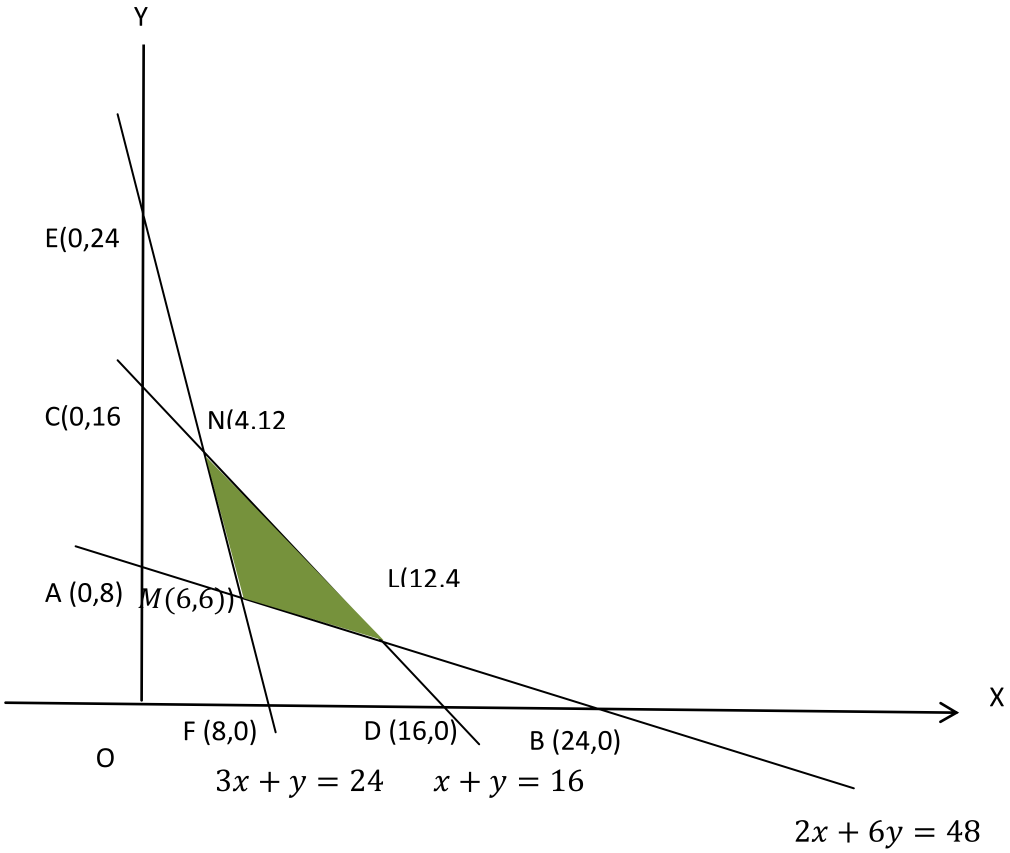
Subject to $x + y \leq 16$

$3x + y \geq 24$

$2x + 6y \geq 48$

$X, y \geq 0$

Now ,we draw the graphs of the linear equations and recognise the feasible region



The shaded region MNL (triangular region ,bounded) is the feasible region.

In this region all the constraints of the problem are satisfied.Now we evaluate the objective function C = $12\ x + 15\ y$ at the verticies.

Vertex	x	y	C = $12x + 15y$
M	6	6	162
N	4	12	228
L	1/2	4	206

From the above table we observe that the minimum value of C is 162 and corresponds to
$x = 6$ and $y = 6$.
Therefore, in order to make the cost minimum 6kg each of fodder-1 and fodder-2 are to be bought and the minimum cost is Rs.162.
Note:- The co-ordinates of the points M, N and L can be read from the graph. They may also be determined by solving the equations of the corresponding pair of lines.