QUESTION BANK II PUC SCIENCE

I. Very Short answer questions. (1x19=19)

1. Define Symmetric relation.

Ans: A relation 'R' on the set 'A' is said to be symmetric if for all a, b, $\in$ A, aR_b Implies bRa. i.e. (a, b) $\in$ R $\Rightarrow$ (b, a) $\in$ R

2. Let A = {4, 6, 8, 20} R = {(a, b); a + b = 25, a, b $\in$ A} Show that the relation 'R' Is empty.

Ans: This is an empty set, as no pair (a, b) satisfies the condition a + b = 25.

3. Give an example of a relation defined on a suitable set which is

i. reflexive, symmetric and transitive.

ii. reflexive, symmetric but not transitive.

iii. reflexive, transitive but not symmetric.

iv. symmetric, transitive but not reflexive.

vi. symmetric, but not reflexive and not transitive.

vii. not reflexive, not symmetric, and not transitive.

Solution: Consider a Set A = {a, b, c}

i. Define a relation R, on 'A' as.
R_1 = {(a, a), (b, b), (c, c)}. Clearly R_1 is reflexive, symmetric and transitive

ii. Consider the relation R_2 on A as.
R_2 = {(a, a), (b, b), (c, c), (a, b), (b, a), (a, c), (c, a)}.
R_2 is symmetric, reflexive
But R_2 is not transitive – for (b, a) $\in$ R_2 and (a, c) $\in$ R_2 but (b, c) $\not\subset$ R_2.

iii. Consider the relation R_3 on A as
R_3 = {(a, a), (b, b), (c, c), (a, b)}
R_3 is reflexive and transitive but not symmetric for (a, b) $\in$ R_3 but (b, a) $\not\subset$ R_3

iv. Consider the relation R_4 on A as
R_4 = {(a, a), (b, b), (c, c), (b, a)} is symmetric and transitive but not reflexive.
Since (c, c) $\not\subset$ R_4.

v. Consider the relation R_5 on A as
R_5 = {(a, a), (b, b), (c, c), (a, b), (c, a)}
R_5 is reflexive. R_5 is not symmetric because (a, b) $\in$ R_5 but (b, a) $\not\subset$ R_5 Also R_5 is not transitive because (c, a) $\in$ R_5, (a, b) $\in$ R_5 but (c, b) $\not\subset$ $R_{5.}$

vi. Consider the relation R_6 on A by
R_6 = {(a, a), (c, c), (a, b), (b, a)}
R_6 is Symmetric but R_6 is not reflexive because (b, b) $\not\subset$ R_6 . Also R_6 is not transitive for (b, a) $\in$ R_6 , (a, b) $\in$ R_6 but (b, b) $\not\subset$ R_6.

vii. The relation R_7 defined on A as
R_7 = {(a, b), (b, c) } is not reflexive, not symmetric and not transitive.

*** FUNCTION * (One mark question and answers)**

4. Let A = {1, 2, 3} B = {4, 5, 6, 7,} and let f = {(1, 4), (2, 5), (3, 6)} be a function from 'A' to 'B'. show that 'f' is not onto.

Ans: 7 $\in$ B has no pre image in A. so 'f' is not onto.

5. If, f : R $\rightarrow$ R is defined by f(x) = 4x-1 $\forall x \in$ R prove that 'f' is one-one.

Solution: For any two elements x_1, x_2 $\in$ R such that $f(x_1) = f(x_2)$
we have $4x_1–1 = 4x_2–1 \Rightarrow 4x_1 = 4x_2$
$\Rightarrow x_1 = x_2$ Hence 'f' is one – one.

6. Define transitive relation.

Ans: A relation R on the set "S" is said to be transitive relation if aRb and bRc $\Rightarrow$ aRc.
i.e. if (a, b) $\Rightarrow$ R and (b, c) $\in$ R $\Rightarrow$ (a, c) $\in$ R.

7. Let f : R $\to$ R be the function defined by f (x) = 2x-3 $\forall x \in R$. write $\overset{-1}{f}$

Ans; y = f(x) = 2x–3 $\Rightarrow$ x = $\frac{1}{2}$ (y + 3)

$\therefore$ $\overset{-1}{f}(y) = \frac{1}{2}$ (y+3) i.e. $\overset{-1}{f}(y) = \frac{1}{2}$ (x+3)

8. Define a binary operation on a set.

Ans: Let S be a non-empty set. The function *: S X S $\to$ S which associates each ordered pair (a, b) of the elements of S to a unique element of S denotes by a*b is called a binary operation or a binary composition on S.

9. Determine whether or not each of the definition defined below is a binary operation justify.
a) on Z^+ defined by a*b = | a-b |
b) on Z^+ defined by a*b = a

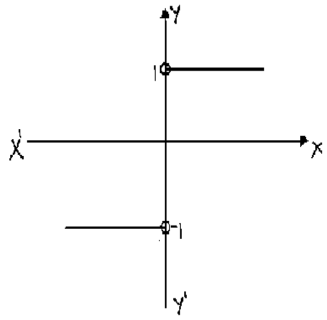
Solution: a) We have for a, b, $\in$ Z^+ a*b = | a – b |. We know the
| a – b | is always positive. $\therefore$ $\forall$ a, b $\in$ Z^+, a*b = | a–b | is a positive integer.
Hence * is a binary operation on Z^+
b) Cleary a*b = $a \in Z^+$ for all $a, b \in Z^+$
Thus * is a binary operation on Z^+

*** SHORT ANSINER QUESTIONS* (Answers to two marks questions)**

10. Define an equivalence relation. Give on example of a relation which is transitive but not reflexive.

Solution: A relation "R" on a Set S is called on equivalence relations. if 'R' is reflexive, symmetric, and transitive.
Ex. The relation "<" (less than) defined on the set R of all real numbers is transitive, but not reflexive.

11. Show that the signum function f : R $\to$ R



defined by $f(x) = \begin{cases} 1 & if \ x > 0 \\ 0 & if \ x = 0 \\ -1 & if \ x < 0 \end{cases}$

is neither one-one nor onto.

Solution: From the graph of the function we have
f(2) = 1 and f(3) = 1, i.e. f(2) = f(3) = 1 but 2 $\neq$ 3
$\therefore$ 'f' is not one-one
Again for 4 $\in$ R (co domain) there exists no $x \in R$ (domain)
Such that f(x) = 4 because f(x) = 1 or –1 for x $\neq$ 0.
$\therefore$ 'f' is not onto.

12. State whether the function f : R $\rightarrow$ defined by f(x) = 1 + x^2 $\forall x \in R$ is one-one, onto or objective justify your answer.

Solution: We have f(x) = 1 + x^2 for all $x \in R$.

clearly we observe that f(x) = 1 + 2^2 = 5 and f(–2) = 1 + $(-2)^2$ = 5 i.e, f(2) = f(–2)

but 2 $\neq$ –2 $\therefore$ 'f' is not one-one.

Again for y –2 there exist no real number x, such that f(x) = –2 because if f(x) = –2 $\Rightarrow$ 1 + x^2 = –2 $\Rightarrow$ x^2 = –3

$\Rightarrow$ x = $\sqrt{-3} \notin R$

$\therefore$ 'f' is not onto.

13. Show that the greatest integer function f : R $\rightarrow$ R defined by f(x) = [x] is neither one-one nor onto.

Solution: We have f(x) = [x] = greatest integer less than or equal to x.

i.e. [2.3] = 2, [2] = 2 but 2.3 $\neq$ 2.

Now we have [x] = 2 for all $x \in$ [2, 3]

$\therefore$ 'f' is not one-one.

Again $\frac{3}{2} \in R$ (codomain), but there existing no $x \in R$ (domain) such that f(x) = $3/2$ because [x] is always an integer $\forall x \in R$

$\therefore$ 'f' is not onto.

14. Cheek the injectivity and surjectivity of the function f : Z $\rightarrow$ Z defined by f(x) = x^3 $\forall x \in Z$

Solution: We have f(x_1) = f(x_2) $\Rightarrow x_1^3 = x_2^3 \Rightarrow x_1 = x_2$

$\therefore$ 'f' is injective.

Now 7 $\in$ Z (codomain) and it has no pre-image in Z because if f(x) = 7 then

$x^3 = 7 \Rightarrow x = \sqrt[3]{7} \notin Z$

$\therefore$ 'f' is not surjective.

15. If f(x) = | x | and g(x) = | 5x – 2 | then find (i) got and (ii) fog.

Solution: We have f(x) = | x | and g(x) = | 5x–2 |

consider go f(x) = g(| x |) = 5 | x | – 2

$$= \begin{cases} |5x-2| & x > 0 \\ |-5x-2| & x < 0 \end{cases}$$

Now fog(x) = f [g(x)] = f(|5x–2 |)

= | 5x–2 |

16. If f ; R $\rightarrow$ R be given by $f(x) = \left(3 - x^3\right)^{1/3}$ find fo f(x).

Solution: We have $f(x) = \left(3 - x^3\right)^{1/3}$

Now to $fof(x) = f\left[f(x)\right] = f\left[\left(3 - x^3\right)^{1/3}\right]$ = f(y)

where $\left(y = (3 - x^3\right)^{1/3}$

$\therefore fof(x) = f(y) = \left(3 - y^3\right)^{1/3}$

$$= \left[3 - \left\{\left(3 - x^3\right)^{1/3}\right\}^3\right]^{1/3} = (x^3)^{1/3} = x.$$

Hence fo f(x) = x.

17. Consider f : {1, 2, 3} $\rightarrow$ {a, b, c} given by f(1) = a, f(2) = b f(3) = C find $\overset{-1}{f}$ and show that $\left(\overset{-1}{f}\right)^{-1} = f$

Solution: Consider f : {1, 2, 3} $\rightarrow$ {a, b, c} given by

f(1) = a, f(2) = b and f(3) = C

$\therefore f(1) = a, f(2) = b$ and f(3)=4

$\overset{-1}{f}(a) = 1, \overset{-1}{f}(b) = 2$ and $\overset{-1}{f}(c) = 3$

$\therefore \overset{-1}{f} = \{(a,1), (b,2), (c,3)\} = g(say)$

we observe that 'g' is also objective.

$\therefore \overset{-1}{g} = \{(1,a), (2,b), (3,c)\} = f$

$\therefore \overset{-1}{g} = f \Rightarrow \left(\overset{-1}{f}\right)^{-1} = f$

18. If f(x) = $= \dfrac{4x+3}{6x-4}, x \neq 2/3$ show that

(fof) (x) = x for all $x \neq 2/3$. what is the inverse f ?

Solution: $(fof)(x) = f[f(x)] = \dfrac{4f(x)+3}{6f(x)-4} = \dfrac{4\dfrac{4x+3}{6x-4}+3}{6.\dfrac{4x+3}{6x-4}-4}$

$= \dfrac{4\ (4x+3)+3(6x-4)}{6\ (4x+3)-4(x-4)} = \dfrac{34x}{34} = x$

$\therefore fof = I \Rightarrow f = \overset{-1}{f}$

19. Let f : R $\rightarrow$ R be defined by f(x) = 10x+7, Find the function g : R $\rightarrow$ R such that gof = fog = Ig

Solution: We have f(x) = 10x+7

By data, gof(x) = Ig(x) $\Rightarrow$ g [f(x)] = x.

$\Rightarrow$ g [10x+7] = x.

Let y = 10x+7 $\Rightarrow$ x $= \left(\dfrac{y-7}{10}\right)$

Then g(10x+7) = $g\left[10\left(\dfrac{y-7}{10}\right)+7\right] = y$

20. Consider a function f : $f : \left[0, \Pi/2\right] \rightarrow R$ given by f(x) = Sinx. and g : $f : \left[0, \Pi/2\right] \rightarrow R$ given by g(x) = cosx. Show that 'f' and 'g' are one-one but (f + g) is not one-one.

Solution: Since for any two distinct elements x_1 and x_2 in $\left[0, \Pi/2\right]$, sin $x_1 \neq$ sin x_2 and cos $x_1 \neq$ cos x_2

$\therefore$ Both 'f' and g are one-one

But (f + g) (o) = f(o) + g(o) = 1 and

$$(f+g)\left(\Pi/2\right) = f\left(\Pi/2\right)+g\left(\Pi/2\right)=\sin \Pi/2+\cos \Pi/2 = 1$$

Therefore (f + g) is not one – one

21. Examine whether the binary operation * defined below are commutative, associative

a. On Q defined loy a * b = ab + 1

b. On Z^+ defined loy a * b = a^b

c. On Q defined loy a * b = ab/2

Solution: a) For every a, b, $\in$ Q we have

i) a * b = ab + 1 $\in$ Q

Now a * b = ab + 1 = ba + 1 (Usual multiplication is commutative)

= b * a

* is commutative in Q.

ii) consider 4, 5, 6 $\in$ Q.

4 * (5 * 6) = 4 * (30 + 1) = 4 * 31)

= 4 (31) + 1 = 124 + 1 = 125

and (4*5) * 6 = 21 * 6 = (21 x 6) + 1 = 157

$\therefore$ a * (b * c) = (a * b) * c is not true for all a, b, c, $\in$ Q. Hence * is not associative.

b) For all a, b, * Z^+, clearly a * b = $a^b \in Z^+$

i.e. * is a binary operation in Z^+

i) If a $\neq$ then $a^b \neq b^a$

i.e. a * b $\neq$ b * a for a $\neq$ b.

Thus * is not commutative in Z^+

ii) Consider 2, 3, 4, $\in Z^+$

(2 * 3) * 4 = 2^3 * 4 = 8 * 4 = $8^4 = 2^{12}$

and 2 * (3 * 4) = 2 * $3^4 = (2)^{34} = 2^{81}$

$\therefore$ (2 * 3) * 4 $\neq$ 2 * (3 * 4)

Thus (a*b)*c = a*(b*c) is not true $\forall a,b,c,\in Z^+$

Hence * is not associative in Z^+.

C. We have a * b = $ab/2$ $\forall$a,b$\in$Q

i) Now a * b = $ab/2 = ba/2$ = b * a

(Usual multiplication is commutative)

$\therefore$ * is commutative in Q.

ii. Now consider a * (b * c) = a * $\left(\frac{bc}{2}\right)$

$$= \frac{a\left(\frac{bc}{2}\right)}{2} = abc/4$$

$$(a*b)*c = \frac{ab}{2}*c = \frac{abc}{4}$$

$\therefore$ (a * b) * c = a * (b * c) for all a, b, c, $\in$ Q.

Hence * is associative in Q.

22. Write the multiplicative modulo 12 table for the set A = {1, 5, 7, 11}. Find the identity element w.r .t X_{12}

X_{12}	1	5	7	11
1	1	5	7	11
5	5	1	11	7
7	7	11	1	5
11	11	7	5	1

Solution: The elements in the row corresponding to the element 1, coincides with the elements of a above the horizontal line in the same order. Thus 1 acts as an identify element.

* ANSIEWERS TO THREE MARKS QUESTIONS*

23. The relation R defined on the Set A = {1, 2, 3, 4, 5} by R = {(a, b) ; | a–b | is even}. Show that the relation 'R' is on equivalence relation. (consider 'O' as an even num ber)

Proof: For any a $\in$ A we have a–a = o considered as even i.e. (a, a,) $\in$ R for all a $\in$ A

Thus R is reflexive relation.

Let (a, b) $\in$ R $\Rightarrow$ |a–b | is even

$\Rightarrow$ |b–a | is also even $\Rightarrow$ (b, a) $\in$ R.

Thus the relation R is symmetric.

Let (a, b) $\in$ R and (b, c) $\in$ R.

$\Rightarrow$ | a–b | is even and | b–c | even $\Rightarrow 1a-b1 = 2k$ and | b – c | =2l for k, l $\in$ Z.

$\Rightarrow$ | a–b | + | b–c | = 2k + 2 l = 2 (k + l)

$\Rightarrow$ a–b + b–c = $\pm$ 2 (n) where n = (k+l)

$\Rightarrow$ a–c = $\pm$ 2n

$\Rightarrow$| a–c | = even $\Rightarrow$ (a, c) $\in$ R

$\therefore$ 'R' Is transitive.

Hence 'R' Is an equivalence relation.

24. Let A = R–{3} and B = R–{1}. Consider the function f : A $\to$ B defined by f(x) = $x-2/_{x-3}$ Is 'f' is one-one and onto ? Justify your answer.

Solution: We have f(x) = $\frac{x-2}{x-3}$ for all x $\in$ R

Now let f(x_1) = f(x_2) $\Rightarrow \frac{x_1-2}{x_2-3} = \frac{x_2-2}{x_2-3}$

$$\Rightarrow x_1x_2 - 2x_2 - 3x_1 + 6 = x_1x_2 - 2x_1 - 3x_2 + 6$$

$$\Rightarrow x_1 = x_2$$

Thus $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$ i.e. 'f' is one-one.

Let $y \in B$ thus $y \neq l.$

Now $f(x) = y \Rightarrow \frac{x-2}{x-3} = y \Rightarrow x-2 = y\ (x-3)$

$\Rightarrow 3y-2 = x\ (y-1)\ \Rightarrow x = \frac{3y-2}{y-1} (\because y \neq 1).$

Also $\frac{2-3y}{1-y} \neq 3-$ for if $\frac{2-3y}{1-y} = 3$

$\Rightarrow 2-3y = 3-3y \Rightarrow 2 = 3$ which is not true.

Thus for every $y \in B$ there exists

$x = \frac{2-3y}{y-1} \in A$ such that $f(x) = y$

i.e. 'f' is onto.

Hence 'f' is bijective function.

25. If 'f' and g are two functions defined by f(x) = 2x + 5 and g(x) = x^2 + 1 find 1 gof(2), ii) fog(2) and iii) gog(2).

Solution: We have $f(x) = 2x+5, g(x) = x^2+1.$

Now $gof(x) = g[f(x)] = g(2x+5)$

$\Rightarrow gof(x) = (2x+5)^2 + 1$

$= 4x^2 + 25 + 20x + 1$

$= 4x^2 + 20x + 26$

$\therefore gof(2) = 4(2)^2 + 20(2) + 26$

$= 16 + 40 + 26 = 82$

ii) $fog(x) = f(x^2+1) = 2(x^2+1) + 5$

$\Rightarrow fog(x) = 2x^2 + 7$

$\therefore fog(2) = 2(2)^2 + 7 = 8 + 7 = 15$

iii) $gog(x) = g(x^2+1)$

$= (x^2+1)^2 + 1 = x^4 + 2x^2 + 2$

$\therefore gog(2) = 16 + 8 + 2 = 26$

26. Let * be the binary operation on N given by a * b = l.c.m of ‘a’ and ‘b’ Is * commutative? Is * associative? Find the identify in N w.r.t *.

Solution: We have a * b = l.c.m. of ‘a’ and ‘b’ $\forall a, b \in N$ clearly l.c.m of two positive integers is a positive integer. Thus N is closed under the operation *.

i) Clearly l.c.m of ‘a’ and b = l.c.m. of ‘b’ and ‘a’

$\therefore$ a*b = b*a for all a, b, $\in$ N

Thus * is commutative.

ii) Let a, b, c, $\in$ N be arbitrary.

Now a*(b * c) = a*(l.c.m. of b and c)

= l.c.m. of [‘a’ and (l.c.m of b and c)]

= l.c.m of (a,b,c) .... (1)

Again, (a * b) * c

= (l.c.m of ‘a’ and b) * c

= l.c.m of [(l.c.m. of ‘a’ and b) and c]

= l.c.m of (a,b,c)................(2)

Thus (a * b) *c = a* (b * c) $\forall a, b, c \in N$

Hence * is associative.

iii) Let e be the identify element in N. Then for all a $\in$ N, we have

a * e = e * a = a

$\Rightarrow \begin{cases} a*e = \text{l.c.m of a and e = a} \\ e*a = \text{l.c.m. of e and a = a} \end{cases}$

l.c.m of a and e = l.c.m. of e and a = a

for all a $\in$ N. implies e = 1.

Thus 1 acts as identify element in N.

*** FIVE MARKS QUESTIONS AND ANSWERS***

27. Consider $f : R^+ \rightarrow [-5, \infty)$ defined by $f(x) = 9x^2 + 6x - 5$ show that ‘f’ is invertible with $\overset{-1}{f}(y) = \sqrt{\frac{y+6-1}{3}}$

Solution: We have $f : R^+ \rightarrow [-5, \infty)$ by $f(x) = 9x^2 + 6x - 5$

Let $a_1, a \in R_+$ Such that $f(a_1) = f(a_2)$

$\Rightarrow 9a_1^2 + 6a_1 - 5 = 9a_2^2 + 6a_2 - 5$

$\Rightarrow 9a_1^2+9a_2^2+6a_1-6a_2=0$

$\Rightarrow 9(a_1-a_2)(a_1+a_2)+6(a_1-a_2)=0$

$\Rightarrow (a_1-a_2)\left[9a_1^2+9a_2^2+6\right]=0$

$\Rightarrow a_1-a_2=0\left(\because 9a_1+9a_2+6\neq o \text{ as } a_1,a_2\in \overset{+}{R}\right)$

$\Rightarrow a_1=a_2$ Thus ‘f’ is one-one.

Let $y=f(x)=9x^2+6x-5$

$\Rightarrow 9x^2+6x-5-y=0$

$\Rightarrow x=\dfrac{-6\pm\sqrt{36+36(y+5)}}{18}$

$\Rightarrow x=\dfrac{-1\pm\sqrt{1+y+5}}{3}=\dfrac{-1\pm\sqrt{y+6}}{3}$

$\therefore \overset{-1}{f}(y)=\dfrac{-1\pm\sqrt{y+6}}{3}$

For every element $y\in B,$ there exists a pre-Image x in$[-5,\infty)$. So ‘f’ is onto Thus ‘f’ is one-one and onto and therefore invertible Hence inverse function of ‘f’ is given by

$\overset{-1}{f}(y)=\dfrac{-1\pm\sqrt{y+6}}{3}$

28. If $f:A\to A$ defined by $f(x)=\dfrac{4x+3}{6x-4}$ where $A=R-\left\{2/3\right\}$. show that ‘f’ is invertible and $\overset{-1}{f}=f$

Solution: We have $f(x)=\dfrac{4x+3}{6x-4}$ with $x\neq 2/3$

Now $f(x_1)=f(x_2)\Rightarrow \dfrac{4x_1+3}{6x_1-4}=\dfrac{4x_2+3}{6x_2-4}$

$\Rightarrow 24x_1x_2-16x_1+18x_2-12=24x_1x_2+18x_1-16x_2-12$

$\Rightarrow 34x_2=34x_1\Rightarrow x_1=x_2$

Thus ‘f’ is one-one.

Now let $y\in A$ and f(x) $f(x)=y$

$\Rightarrow y\in A$ and $\dfrac{4x+3}{6x-4}=y$

$\Rightarrow y\in A$ and $x=\dfrac{4y+3}{6x-3}$

Thus for every $y\in A$, there exists

$x=\dfrac{4y+3}{6y-3}\in A\left(\because y\neq 2/3\right)$ such that

$f(x)=y.$ That is ‘f’ is onto.

Hence ‘f’ is bijective.

Now $\overset{-1}{f}(x)=y\Rightarrow f(y)=x$

$\Rightarrow \frac{4y+3}{6x-4} = x \Rightarrow y = \frac{4x+3}{6x-4}$

Thus $\overset{-1}{f} : A \to A$ is defined by $\overset{-1}{f}(x) = \frac{4x+3}{6x-4}$

Clearly $\overset{-1}{f} = f$.

29. Let f; w $f : w \to w$ be defined by

$$f(n) = \begin{cases} n-1 & \text{n is odd} \\ n+1 & \text{if 'n' iseven} \end{cases}$$

Show that 'f' is invertible and find

The inverse of 'f'. Hence w is the set of all whole numbers.

Solution: Let $x_1, x_2 \in$ w be arbitrary

Let x_1 and x_2 are even.

$f(x_1) = f(x_2) \Rightarrow 1 + x_1 = 1 + x_2 \Rightarrow x_1 = x_2$

Let x_1 and x_2 are odd

$f(x_1) = f(x_2) \Rightarrow x_1 - 1 = x_2 - 1 \Rightarrow x_1 = x_2$

Let x_1 and x_2 are odd $f(x_1) = f(x_2) \Rightarrow x_1 - 1 = x_2 - 1$

$\Rightarrow x_1 = x_2$

In both the cases, $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$

Supposing x_1 is odd and x_2 is even

then $x_1 \neq x_2$. Now $f(x_1) = x_1 - 1$and $f(x_2) = x_2 - 1$

Also $x_1 - 1 \neq x_2 - 1$

i.e. $x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$

Hence 'f' is one-one

Let $m \neq 0 \in w$ (co domain) and f(n) = m

Now $f(n) = m \Rightarrow f(n) = \begin{cases} n-1 = m & \text{if 'n' is odd} \\ n+1 = m & \text{if 'n' is even.} \end{cases}$

$$\Rightarrow f(n) = \begin{cases} n = m+1 & \text{if 'n' is odd} \\ n = m-1 & \text{if 'n' is even.} \end{cases}$$

Thus for every $m \in w$ (co domain)

there exists $n \in w$ (= m+1 or m–1)

Such that f(n) = m.

Also (considering O as even)

$f(o) = 0 + 1 = 1$ i.e. $\overset{-1}{f}(1) = 0$

Thus 'f' is onto

Hence 'f' is a bijection.

Let $\overset{-1}{f}(n) = m \Rightarrow f(m) = n.$

$$\Rightarrow f(m) = \begin{cases} m-1 = n & {}^{-}m \text{ is odd} \\ m+1 = n & {}^{-}m \text{ is even} \end{cases}$$

$$\Rightarrow f(m) = \begin{cases} m = n+1 & {}^{-}n \text{ is even} \\ m = n-1 & {}^{-}n \text{ is odd} \end{cases}$$

Thus $f^{-1}(n)=\begin{cases} n+1 & {}^{-}n \text{ is even} \\ n-1 & {}^{-}n \text{ is odd} \end{cases}$

Is inverse function.