2. ELECTROSTATIC POTENTIAL AND CAPACITANCE

1) **What do you mean by the conservative nature of electric field?**
The conservative nature of electric field means that the work done to move a charge from one point to another point in electric field is independent of path, but it depends only on the initial and final positions of the charge.

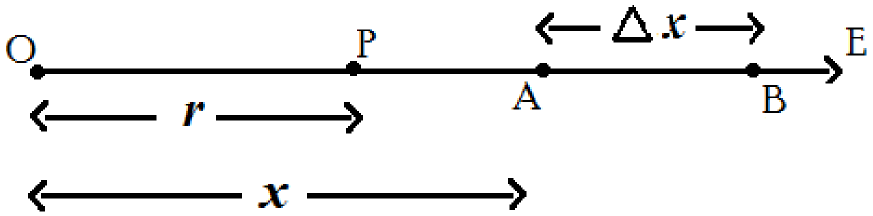
2) **Define Electrostatic Potential.**
Electrostatic potential at a point in field is defined as the work done to move a unit positive charge, without any acceleration from infinity to that point at consideration.
If W is the work done to move a charge q from infinity to a point, then the electric potential at that point is

$$V = \frac{W}{q}$$

SI Unit is J C^{-1} = volt (V)

3) **What is the SI unit of Electric Potential?**
SI Unit of Electric Potential is J C^{-1} = volt (V)

4) **Derive the expression for electric potential at a point due to a point charge.**
Consider a point charge Q at origin O. Let P be a point at a distance *r* from it. Let A and B be two points at a distance x and $x + \Delta x$ from O along the line OP.



The force experienced by unit positive charge (+1C) at A

$$\vec{F} = \frac{1}{4\pi\varepsilon_o} \frac{QX1}{x^2} \hat{x} \qquad \ldots\ldots \quad (1)$$

Where $\hat{x}$ is unit vector along OA

The work done to move a unit positive charge (+1C) from B to A is

$$\Delta W = \vec{F} \,.\, \overrightarrow{\Delta x} = F\,\Delta x \cos\theta$$

Here, $\vec{F}$ and $\overrightarrow{\Delta x}$ are opposite, therefore $\theta = 180^0$

$$\Delta W = F\,\Delta x \cos 180 = F\,\Delta x\,(-1)$$

$$\Delta W = -F\,\Delta x$$

From eq (1), we get

$$\Delta W = -\frac{1}{4\pi\varepsilon_o}\frac{QX1}{x^2}\Delta x$$

The potential at a point P is work done to move unit positive charge (+1C) from infinity to P, therefore

$$V = \int_{\infty}^{r} -\frac{1}{4\pi\varepsilon_o}\frac{QX1}{x^2}\,dx = -\frac{Q}{4\pi\varepsilon_o}\int_{\infty}^{r}\frac{1}{x^2}\,dx$$

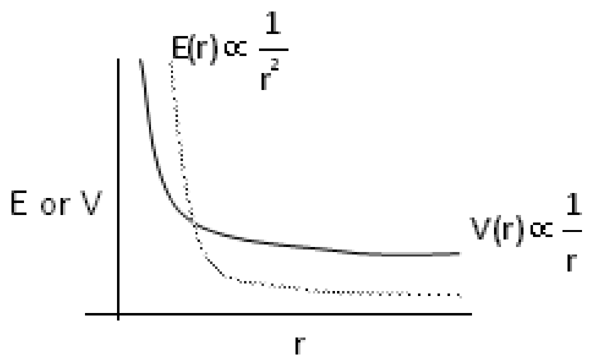
$$V = -\frac{Q}{4\pi\varepsilon_o}\left(\frac{-1}{x}\right)_{\infty}^{r} = \frac{Q}{4\pi\varepsilon_o}\frac{1}{r}$$

$$V = \frac{1}{4\pi\varepsilon_o}\frac{Q}{r}$$

5) **How does the electric field and electric potential vary with distance from a point charge?**

The electric field $\overrightarrow{E} = \frac{1}{4\pi\varepsilon_o}\frac{Q}{r^2}\hat{r} \Rightarrow E\ \alpha\ \frac{1}{r^2}$

The electric potential $V = \frac{1}{4\pi\varepsilon_o}\frac{Q}{r} \Rightarrow V\ \alpha\ \frac{1}{r}$



6) **Write the expression for electric potential at a point due to an electric dipole and hence obtain the expression for the same at any point on its axis and any point on its equatorial plane.**

$$V = \frac{1}{4\pi\varepsilon_o}\left(\frac{p\cos\theta}{r^2}\right)$$

Where $p = 2\,aq$ is the electric dipole moment.

r = distance of the point from the centre of the dipole.

θ = the angle between $\vec{p}$ and $\vec{r}$.

For any point on the dipole axis, $\theta = 0\ or\ \pi \Rightarrow \cos\theta = \pm 1$, we get

$$V = \pm\frac{1}{4\pi\varepsilon_o}\left(\frac{p}{r^2}\right)$$

For any point on the equatorial plane, $\theta = \frac{\pi}{2} \Rightarrow \cos\theta = 0$, we get

$$V = 0$$

7) How does the electric potential at a point due to an electric dipole vary with distance measured from its centre? Compare the same for a point charge.

For an electric dipole (at large distances), we have

$$V = \frac{1}{4\pi\varepsilon_o}\left(\frac{p\cos\theta}{r^2}\right) \Rightarrow V\ \alpha\ \frac{1}{r^2}$$

The electric potential varies inversely with the square of the distance.

For a point charge,

$$V = \frac{1}{4\pi\varepsilon_o}\left(\frac{q}{r}\right) \Rightarrow V\ \alpha\ \frac{1}{r}$$

The electric potential varies inversely with the distance.

8) Using superposition principle, write the expression for electric potential at a point due to a system of charges.

$$V = \frac{1}{4\pi\varepsilon_o}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + \cdots\ \frac{q_n}{r_n}\right)$$

Where $q_1, q_2, q_3 \ldots\ldots q_n$ are the point charges and $r_1, r_2, r_3 \ldots\ldots r_n$ are the distances of the point from the respective point charges.

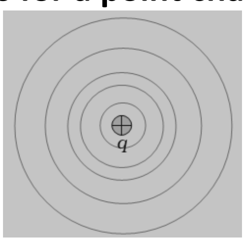
9) What is an equipotential surface? Give an example.

An Equipotential surface is a surface with same potential at all points on it. The surface of a charged conductor is an example.

10) What are the equipotential surfaces of a point charge?

The Equipotential surfaces of a point charge are the concentric spheres with centre at the point charge.

11) Draw the Equipotential surfaces for a point charge.



12) Give the condition for equipotential surface in terms of the direction of the electric field.

The electric field is always perpendicular to the equipotential surface.

13) Explain why the equipotential surface is normal to the direction of the electric field at that point.

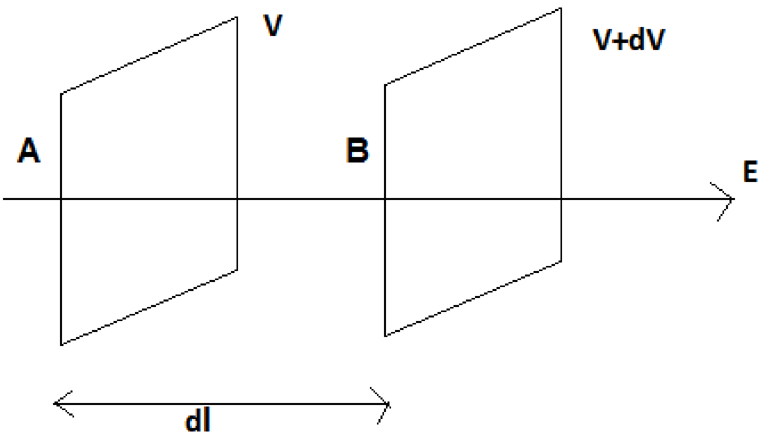
If the Equipotential surface is not normal to the direction of the electric field at a point, then electric field will have a non- zero components along the surface and due to this work must be done to move a unit positive charge against this field component. This means that there is potential difference between two points on the surface. This contradicts the definition of equipotential surface.

14) **Obtain the relation between the electric field and potential.**

OR

Show that electric field is in the direction in which the potential decreases steepest.

Consider two equipotential surfaces A and B with the potential difference dV between them as shown in figure. Let dl be the perpendicular distance between them and $\vec{E}$ be the electric field normal to these surfaces.



The work done to move a unit positive charge from B to A against the field $\vec{E}$ through a displacement $\overrightarrow{dl}$ is

$$dW = \vec{E}\,.\overrightarrow{dl} = E\,dl\cos\pi = -E\,dl$$

This is equal to the potential difference , therefore

$$dV = dW \;\Rightarrow\; dV = -E\,dl$$

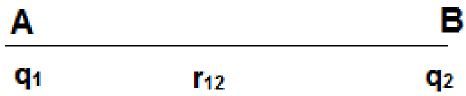
$$E = -\frac{dV}{dl}$$

15) **Define Electrostatic Potential energy of a system of charges.**

Electrostatic potential energy of a system of charges is defined as the work done to move the charges from infinity to their present configuration.

16) **Derive the expression for potential energy of two point charges in the absence of external electric field.**

Consider a system of two point charges $q_1,\ q_2$ separated by a distance r_{12}, as shown in figure.



The work done to move q_1 from infinity to A is,

$$W_1 = 0 \qquad (\because \text{There is no initial electric field})$$

The work done to move q_2 from infinity to B is,

$$W_2 = V_{1B} q_2$$

Where V_{1B} is the electric potential at B due to q_1, it is given by

$$V_{1B} = \frac{1}{4\pi\varepsilon_o} \left(\frac{q_1}{r_{12}}\right)$$

$$\Rightarrow \; W_2 = V_{1B} q_2 = \frac{1}{4\pi\varepsilon_o} \left(\frac{q_1}{r_{12}}\right) q_2$$

$$W_2 = \frac{1}{4\pi\varepsilon_o} \left(\frac{q_1 q_2}{r_{12}}\right)$$

The potential energy of this system of charge is equal to total work done to build this configuration. Therefore

$$U = W_1 + W_2 = 0 + \frac{1}{4\pi\varepsilon_o} \left(\frac{q_1 q_2}{r_{12}}\right)$$

$$\Rightarrow \; U = \frac{1}{4\pi\varepsilon_o} \left(\frac{q_1 q_2}{r_{12}}\right)$$

17) **Write the expression for potential energy of two point charges in the absence of external electric field.**

Electrostatic potential energy is

$$U = \frac{1}{4\pi\varepsilon_o} \left(\frac{q_1 q_2}{r_{12}}\right)$$

Where q_1, q_2 are the point charges and r_{12} is the distance between them.

18) **Write the expression for potential energy of two point charges in the presence of external electric field.**

Electrostatic potential energy is

$$\Rightarrow\ U = q_1V_1 + q_2V_2 + \frac{1}{4\pi\varepsilon_o}\left(\frac{q_1q_2}{r_{12}}\right)$$

Where $q_1,\ q_2$ are the point charges, V_1 and V_2 are potentials at the positions of $q_1,\ q_2$ respectively and r_{12} is the distance between them.

19) Mention the expression for potential energy of an electric dipole placed in an uniform electric field. Discuss its maximum and minimum values.

$$U = -p\,E\cos\theta = -\vec{p}\,.\vec{E}$$

Where p = dipole moment.
E = Electric field.
θ = Angle between dipole axis and electric field.

When the dipole axis is perpendicular to the field

$$\theta = 90 \quad \Rightarrow \quad U = 0 \quad (\,minimum)$$

P E is minimum (zero).
When the dipole axis is parallel to the field

$$\theta = 90 \quad \Rightarrow \quad U = 0 \quad (\,maximum)$$

P E is maximum.

20) Explain why Electric field inside a conductor is always zero.

Otherwise free electrons would experience force and drift causing electric current.

21) Explain why Electrostatic field is always normal to the surface of charged conductor.

If $\vec{E}$ is not normal, it will have component parallel to the surface causing surface currents.

22) Explain why Electric charges always reside on the surface of a charge conductor.

Because, If there are static charges inside the conductor, Electric field can be present inside it which is not true.

23) Explain why Electrostatic potential is constant throughout the volume.

Because the Electric field inside the conductor is zero, therefore no work is done to move a charge against field and there is no potential difference between any two points. This means Electrostatic potential is constant throughout the volume.

24) Write the expression for Electric field near the surface of a charge conductor.

Electric field at the surface of a charged conductor is

$$\vec{E} = \frac{\sigma}{\varepsilon_o}\hat{n}$$

$\hat{n}$ is unit vector normal to the surface.

25) **What is Electrostatic shielding? Mention one use of it.**

The field inside the cavity of a conductor is always zero and it remains shielded from outside electric influence. This is known as electrostatic shielding. This property is used in protecting sensitive instruments from outside electrical influence.

26) **What are Dielectrics? Mention the types of Dielectrics.**

Dielectrics are non-conducting substances. They have no charge carriers.
There are two types of Dielectrics, polar Dielectrics and non-polar Dielectrics.

27) **What are non-polar Dielectrics? Give examples.**

The non-polar Dielectrics are those in which the centers of positive and negative charges coincide. These molecules then have no permanent (or intrinsic) dipole moment.
Examples of non-polar molecules are oxygen (O_2) and hydrogen (H_2).

28) **What are polar Dielectrics? Give examples.**

The non-polar Dielectrics are those in which the centers of positive and negative charges are separated (even when there is no external field). Such molecules have a permanent (or intrinsic) dipole moment.
Examples of non-polar molecules are HCl and molecule of water (H_2O).

29) **What happens when Dielectrics are placed in an electric field?**

Both polar and non-polar dielectrics develop a net dipole moment along the external field when placed in it.

30) **What is electric polarisation?**

When Dielectric is placed in an external electric field, a net dipole moment is developed along it. The dielectric is now said to be polarised.
The dipole moment acquired per unit volume is known as polarisation $\vec{P}$.

31) **Define capacitance of a capacitor. What is SI unit?**

Capacitance of a capacitor is defined ate ratio of the charge Q on it to the potential difference V across its plates.

$$C = \frac{Q}{V}$$

SI unit is $C\,V^{-1} = farad\ (F)$.

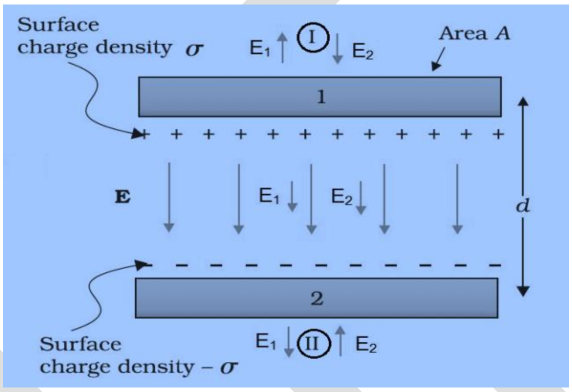
32) What is Parallel plate capacitor?

Parallel plate capacitor is a capacitor with two identical plane parallel plates separated by a small distance and the space between them is filled by an dielectric medium.

33) Derive the expression for capacitance of a Parallel plate capacitor without any dielectric medium between the plates. *(or Parallel plate air capacitor).*

Consider a parallel plate capacitor without any dielectric medium between the plates.

Let A be the area of the plates and d be the plate separation.



We know that the electric field due to uniformly charged plate is E = $\frac{\sigma}{2\varepsilon_0}$

Where σ is the surface charge density of plates.

The electric field in outer region I : $E = E_1 - E_2 = \frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0} = 0$

The electric field in outer region II : $E = E_1 - E_2 = \frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0} = 0$

The electric field between the plates:

$$E = E_1 + E_2 = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$$

But, $\sigma = \frac{Q}{A}$, therefore

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0}$$

We have $E = \frac{V}{d}$, therefore,

$$\frac{V}{d} = \frac{Q}{A\varepsilon_0} \quad \Rightarrow \quad V = \frac{Qd}{A\varepsilon_0}$$

The capacitance is

$$C = \frac{Q}{V}$$

Therefore,

$$C = \frac{Q}{\left(\frac{Qd}{A\varepsilon_0}\right)}$$

$$C = \frac{\varepsilon_0 A}{d}$$

34) Mention the expression for capacitance of a Parallel plate capacitor without any dielectric medium between the plates.

$$C = \frac{\varepsilon_0 A}{d}$$

Where ε_0 = Permittivity of free space.
A = Area of the plates.
D = Plate separation.

35) Mention the expression for capacitance of a Parallel plate capacitor with a dielectric medium between the plates.

$$C = \frac{\varepsilon_0 K A}{d}$$

Where ε_0 = Permittivity of free space.
K = dielectric constant of the medium between the plates.
A = Area of the plates.
D = Plate separation.

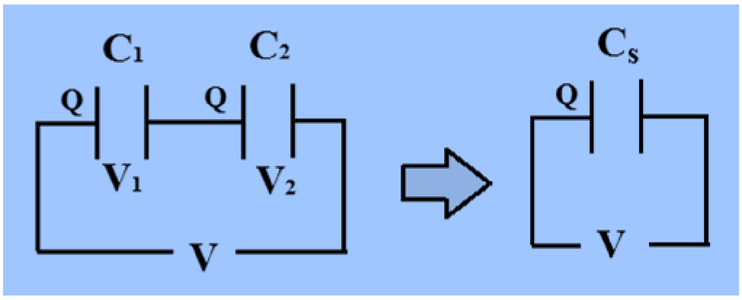
36) Define Dielectric constant of a substance.

Dielectric constant of a substance is defined as the ratio of the permittivity of the medium to the permittivity of free space.

$$K = \frac{\varepsilon}{\varepsilon_0}$$

37) Derive the expression for effective capacitance of two capacitors connected in series.

Consider two capacitors of capacitance C_1 and C_2 connected in series.



In series combination charge Q is same on each capacitor and potential V across the combination is equal to sum of the potential across each. Therefore

$$V = V_1 + V_2$$

But

$$C_1 = \frac{Q}{V_1} \Rightarrow V_1 = \frac{Q}{C_1}, \quad similarlly \;\; V_2 = \frac{Q}{C_2}$$

$$V = \frac{Q}{C_1} + \frac{Q}{C_2}$$

If C_s is the effective capacitance of the series combination, then

$$C_s = \frac{Q}{V} \Rightarrow V = \frac{Q}{C_s}$$

Therefore,

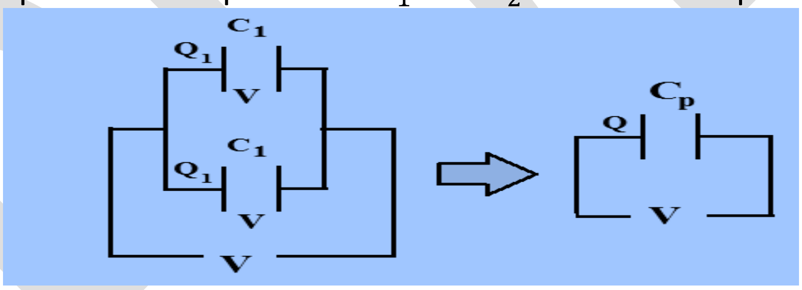
$$\frac{Q}{C_s} = \frac{Q}{C_1} + \frac{Q}{C_2}$$

Or

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2}$$

38) Derive the expression for effective capacitance of two capacitors connected in parallel.

Consider two capacitors of capacitance C_1 and C_2 connected in parallel.



In parallel combination potential V is same across each capacitor and Charge Q on the combination is equal to sum of the charges on each. Therefore

$$Q = Q_1 + Q_2$$

But

$$C_1 = \frac{Q_1}{V} \Rightarrow Q_1 = VC_1, \quad similarlly \;\; Q_2 = VC_2$$

$$Q = VC_1 + VC_2$$

If C_P is the effective capacitance of the parallel combination, then

$$C_P = \frac{Q}{V} \Rightarrow Q = VC_P$$

Therefore,

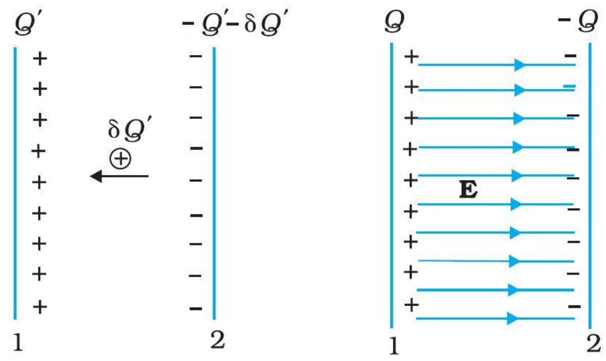
$$VC_P = VC_1 + VC_2$$

$$C_P = C_1 + C_2$$

39) Derive the expression for energy stored in a capacitor.

Consider a capacitor of capacitance C.

At any intermediate stage of charging, let Q' and $-Q'$ be the charge on positive and negative plate respectively and V' be the p.d across the plates.



The work done dW to move small charge dQ' from $-ve$ plate to $+ve$ plate is

$$dW = V' \, dQ'$$

But,

$$C = \frac{Q}{V'} \Rightarrow V' = \frac{Q}{C}$$

Therefore,

$$dW = \frac{Q}{C} \, dQ'$$

The total work done to transfer Q charge from $-ve$ to $+ve$ plate

$$W = \int_0^Q dW = \int_0^Q \frac{Q}{C} \, dQ$$

$$W = \frac{Q^2}{2C} = \frac{1}{2}\frac{Q^2}{C}$$

The work done is stored as energy U in the charged capacitor.

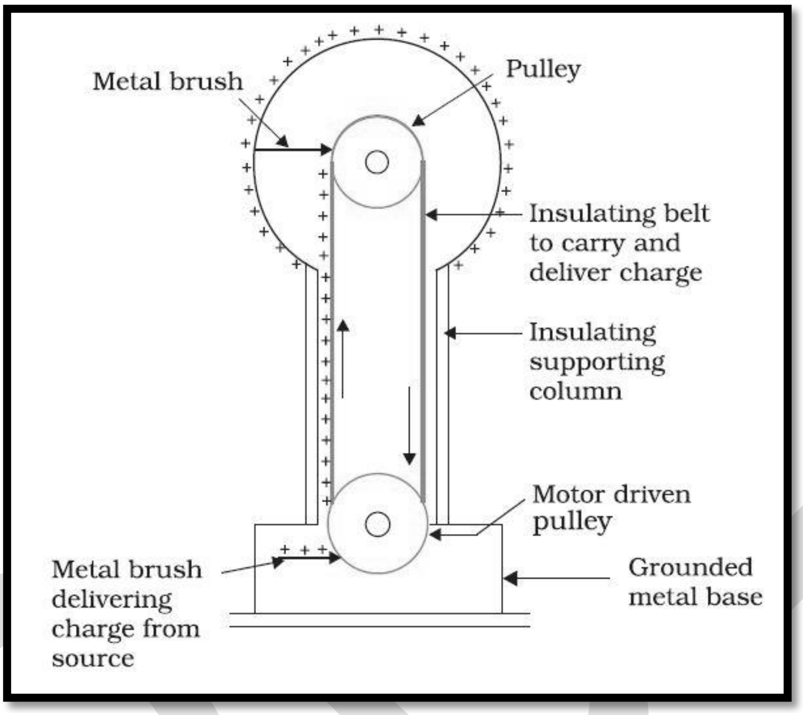
Therefore,

$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}QV = \frac{1}{2}CV^2$$

40) What is Van De Graff generator? Write its labeled diagram. What is the principle of its working? Mention its use.

Van De Graff generator is a machine which generates very high voltages of the order of 10^6 V.

<u>Labeled diagram:</u>



<u>Principle;</u> A small conducting sphere when placed inside a large spherical shell is always at higher potential irrespective of the charge on the outer shell. Thus the charge supplied to the inner sphere always rushes to the outer shell building very high voltages.

<u>Use</u>; The high voltage generated in Van De Graff generator is used to accelerate charged particles *(Particle accelerators).*

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