

13. NUCLEI

One Mark Questions

1. How many neutrons will be there in the nucleus of an element with mass number A and atomic number Z?

A-Z

2. Mention the commonly used unit to measure the nuclear mass.

Atomic Mass Unit denoted by amu or u.

3. Which type of radioactive emission produces a daughter nucleus which is an isobar of the parent?

Beta particle

4. Mention the SI unit of activity.

Becquerel (Bq)

5. What are isotones?

Nuclei of different elements having same number of neutrons

6. How does the radius of the nucleus vary with respect to mass number?

$$R = R_0 A^{1/3}$$

Two Mark Questions

7. What is mass defect? Write the formula for the mass defect for the nucleus of an element ${}^A_Z X$

The difference between the sum of the masses of the constituent particles and the actual mass of the nucleus is known as mass defect.

$$\text{Mass defect } \Delta M = \{Zm_p + (A - Z)m_n\} - M$$

8. Mention any two characteristics of nuclear forces.

Strongest force in nature / short range / non-central / charge independent / spin dependent / saturated – any two

9. Mention the order of nuclear density. How does the nuclear density vary as we move from the centre to the surface?

$$\rho \approx 10^{17} \text{ kg / m}^3 ; \text{ nuclear density remains constant .}$$

10. Define nuclear fission and give an example for it.

Process of splitting up of heavy nucleus into two or more fragments of comparable masses along with the liberation of energy is known as nuclear fission.



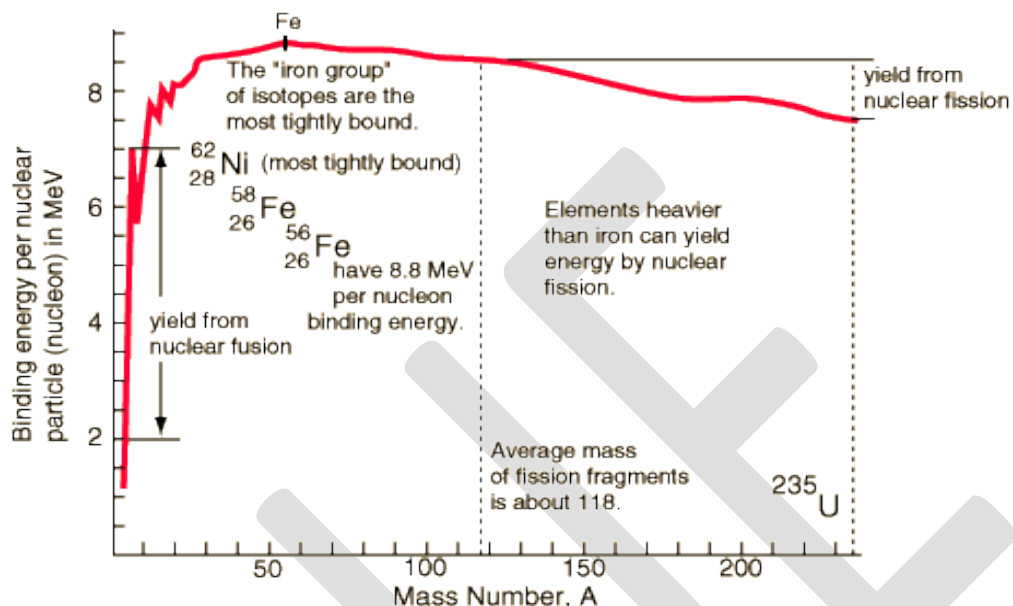
11. Define half-life and mean-life of a radioactive nucleus.

Time during which the number of radioactive atoms will reduce to half the original number is known as half-life of radioactive element.

Average life expectancy of the nucleus is called mean-life. It is the average life of all the atoms which will disintegrate anywhere between zero and infinity. It is numerically equal to time during which number of atoms reduced to about 37% of the original number.

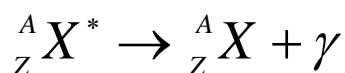
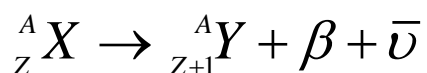
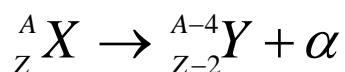
Three Mark Questions

12. Draw the graph of binding energy per nucleon with respect to mass number. What is the significance of the graph?



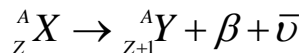
It represents the stability of the nucleus.

13. Write the equation representing nuclear reaction corresponding to α , β and γ emission.



14. What is Q-value of a nuclear reaction? Write the formula for Q-value for β -emission and explain the terms.

During alpha or beta emission total mass of the products is slightly less than the reactant nucleus. The difference in the mass will be converted into energy which will appear in the form of kinetic energy of products- the sum of the kinetic energies of the emitted particle(s) and the recoil nucleus. The energy equivalent of mass difference is known as Q-value . For β -emission



$$Q = (m_x - m_y - m_\beta - m_{\bar{\nu}}) \cdot C^2 ;$$

Where,

$m_x \rightarrow$ mass of the parent nucleus

$m_y \rightarrow$ mass of the daughter nucleus

$m_\beta \rightarrow$ mass of the β - particle

$m_{\bar{\nu}} \rightarrow$ mass of the antineutrino

and C is the speed of light

15. Define Atomic Mass Unit. Mention Einstein's mass energy relation.

$1 / 12^{\text{th}}$ the mass of one atomic nucleus of carbon-12 is known as Atomic Mass Unit denoted by amu or u. This unit is normally used to measure the mass of the nuclei.

According to Einstein's mass energy relation, when a mass 'm' is converted into energy, the energy equivalent E is given by

$$E = mC^2$$

NOTE: Energy equivalent of one amu

We know that

$$\text{Mass of one mole of C-12} = 12\text{g.} = 12 \times 10^{-3} \text{kg.}$$

$$\text{Therefore, mass of 1 atom of C-12} = \frac{12 \times 10^{-3}}{6.023 \times 10^{23}} \text{kg.}$$

$$\begin{aligned} \therefore \text{Energy equivalent of 1amu} &= \Delta M \cdot C^2 \\ &= \frac{12 \times 10^{-3}}{6.023 \times 10^{23}} \cdot (3 \times 10^8)^2 \text{ J} \\ &= \frac{12 \times 10^{-3} \cdot (3 \times 10^8)^2}{6.023 \times 10^{23} \times 1.6 \times 10^{-19}} \text{ eV} \\ &\cong 931 \times 10^6 \text{ eV} = 931 \text{ MeV} \end{aligned}$$

16. Prove that $N = N_0 e^{-\lambda t}$ where the symbols have their usual meaning.

According to radioactive decay law, rate of disintegration of a radioactive substance is directly proportional to the number of radioactive atoms present at that instant of time. Therefore,

$$\frac{dN}{dt} = -\lambda N$$

$$\Rightarrow \frac{dN}{N} = -\lambda dt$$

$$\Rightarrow \int \frac{dN}{N} = \int -\lambda dt$$

$$\Rightarrow \log N = -\lambda t + C$$

$$\text{At } t = 0, N = N_0$$

$$\Rightarrow \log N = -\lambda t + \log N_0$$

$$\Rightarrow \log \frac{N}{N_0} = -\lambda t$$

$$\Rightarrow \frac{N}{N_0} = e^{-\lambda t}$$

$$\therefore N = N_0 e^{-\lambda t}$$

17. Define mean-life. Write the expression for mean-life in terms of decay constant..

The average life or mean-life of a radioactive sample is the ratio of total life time of all N_0 number of atoms in the sample to the total number of atoms which will disintegrate anywhere between zero and infinity. Therefore,

$$\text{mean-life } \tau = \frac{\text{life-time of all atoms}}{N_0}$$

$$\tau = \frac{1}{\lambda}$$

18. Obtain the relation between half-life and decay constant.

Relation between half-life and decay constant

We know that,

$$N = N_0 e^{-\lambda t}$$

$$\text{When } t \rightarrow T \text{ then } N \rightarrow \frac{N_0}{2}$$

$$\Rightarrow \frac{N_0}{2} = N_0 e^{-\lambda T}$$

$$\Rightarrow e^{-\lambda T} = \frac{1}{2}$$

$$\Rightarrow e^{\lambda T} = 2 \Rightarrow \lambda T = \ln 2 = 0.693$$

$$\Rightarrow T = \frac{0.693}{\lambda} = 0.693\tau$$

20) Calculate the binding energy and binding energy per nucleon in MeV for carbon-12 nucleus. Given that mass of the proton is 1.00727amu while the mass of the neutron is 1.00866amu.

Mass defect

$$\Delta M = Zm_p + (A - Z)m_n - M$$

For carbon-12 nucleus

$$Z = 6, A = 12 \text{ and } M = 12 \text{amu}$$

$$\Rightarrow \Delta M = 6 \times 1.00727 + 6 \times 1.00866 - 12 = 0.09558 \text{amu}$$

$$E = \Delta M \times 931 \text{MeV} = 0.09558 \times 931 = 88.985 \text{MeV}$$

21) Half-life of ${}_{38}^{90}\text{Sr}$ is 28years. Calculate the activity in Ci of 30mg of ${}_{38}^{90}\text{Sr}$.

90g of ${}_{38}^{90}\text{Sr}$ contains 6.023×10^{23} atoms

\therefore 30mg of ${}_{38}^{90}\text{Sr}$ contains $\frac{30 \times 10^{-3}}{90} \times 6.023 \times 10^{23} = 2.0077 \times 10^{20}$ atoms

$$\text{Activity } A = \lambda N = \frac{0.693}{T} \cdot N$$

$$\Rightarrow A = \frac{0.693}{28 \times 365 \times 24 \times 3600} \times 2.0077 \times 10^{20} = 1.5757 \times 10^{11} \text{ Bq}$$

$$1\text{Ci} = 3.7 \times 10^{10} \text{ Bq}$$

$$\therefore A = \frac{1.5757 \times 10^{11}}{3.7 \times 10^{10}} = 4.26\text{Ci}$$

22) Calculate the Q-value of the emitted α -particle in the α -decay of ${}_{86}^{220}\text{Rn}$.

Given

Mass of ${}_{86}^{220}\text{Rn} = 220.01137\text{amu}$

Mass of residual nucleus ${}_{84}^{216}\text{Po} = 216.00189\text{amu}$

Mass of α -particle = 4.002603amu

$$\begin{aligned} Q &= (m_{\text{Rn}} - m_{\text{Po}} - m_{\alpha}) \cdot C^2 J = (m_{\text{Rn}} - m_{\text{Po}} - m_{\alpha}) \cdot 931\text{MeV} \\ &= (220.01137 - 216.00189 - 4.002603) \times 931 = 6.402\text{MeV} \end{aligned}$$
