1. ELECTRIC CHARGES AND FIELDS

One mark questions with answers

1. What are point charges?

A: Charges whose sizes are very small compared to the distance between them are called point charges

2. The net charge of a system of point charges $-4, +3$, -1 & $+4$ (S.I.units) =? A: $+2$

3. What is meant by conservation of charge?

A: The total charge of an isolated system remains always constant

4. What is quantisation of charge?

A: The electric charge is always an integral multiple of 'e' (charge on an electron).

5. Mention the S.I. unit of charge. A: coulomb (C)

6. Define one coulomb of charge.

A: 1C is the charge that when placed at a distance of 1m from another charge of the same magnitude, in vacuum, experiences an electrical force of repulsion of magnitude $9\text{x}10^9$N

7. Which principle is employed in finding the force between multiple charges? A: Principle of superposition

8. Define electric field.

A: Electric field due to a charge at a point in space is defined as the force experienced by a unit positive charge placed at that point.

9. Is electric field a scalar/vector? A: vector

10. Mention the S.I. unit of electric field. A: newton per coulomb (NC^{-1})

11. What is the direction of electric field due to a point positive charge? A: Radially outward

12. What is the direction of electric field due to a point negative charge? A: Radially inward

13. What is a source charge? A: The charge which produces the electric field

14. What is a test charge? A: The charge which detects the effect of the source charge

15. How do you pictorially map the electric field around a configuration of charges? A: Using electric field lines

16. What is an electric field line?

A: An electric field line is a curve drawn in such a way that the tangent to it at each point represents the direction of the net field at that point

17. What is electric flux?

A: Electric flux over a given surface is the total number of electric field lines passing through that surface.

18. Mention the S.I.unit of electric flux. A: $Nc^{-1}m^2$

19. What is an electric dipole?

A: An electric dipole is a set of two equal and opposite point charges separated by a small distance

20. What is the net charge of an electric dipole? A: zero

21. Define dipole moment.

A: Dipole moment of an electric dipole is defined as the product of one of the charges and the distance between the two charges.

22. Is dipole moment a vector / scalar? A: Vector

23. What is the direction of dipole moment?

A: The dipole moment vector is directed from negative to positive charge along the dipole axis

24. What is the net force on an electric dipole placed in a uniform electric field? A: Zero

25. When is the torque acting on an electric dipole placed in a uniform electric field maximum?

A: When the dipole is placed perpendicular to the direction of the field

26. When is the torque acting on an electric dipole placed in a uniform electric field minimum?

A: When the dipole is placed parallel to the direction of the field

27. State Gauss's law.

A: Gauss's law states that 'the electric flux through a closed surface is equal to $\frac{1}{\varepsilon_0}$ times the charge enclosed by that surface'

28. What is a Gaussian surface?

A: The closed surface we choose to calculate the electric flux and hence to apply Gauss's law

29. What happens to the force between two point charges if the distance between them is doubled? A: Decreases 4 times.

30. If two charges kept in 'air' at a certain separation, are now kept at the same separation in 'water' of dielectric constant 80, then what happens to the force between them?

A: Decreases by 80 times.

31. On a macroscopic scale is charge discrete or continuous? A: Continuous.

Two mark questions with answers

1. Write the expression for quantisation of charge and explain the terms in it.

A: $q=ne$; n is an integer (+ or-) and e is the charge on an electron

2. State and explain Coulomb's law of electrostatics.

A: Coulomb's law states that 'the electrical force between two point charges is directly proportional to the product of their strengths and is inversely proportional to the square of the distance between them'.

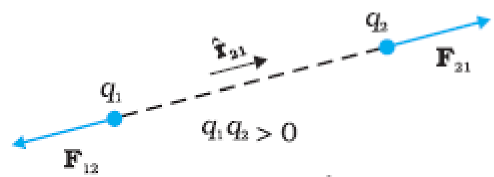
If 'F' represents the electrical force between two point charges q_1 and q_2 separated by a distance 'r' apart, then according to this law $\boxed{F = k\,\frac{q1q2}{r^2}}$; $k = \frac{1}{4\pi\varepsilon_0}$ is a constant when the two charges are in vacuum.

3. Write Coulomb's law in vector notation and explain it.

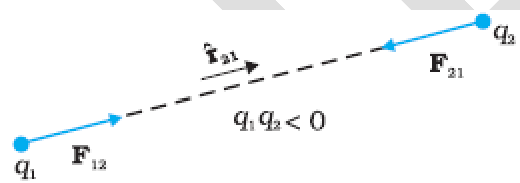
A: $\boxed{\mathbf{F}_{21} = \frac{1}{4\pi\varepsilon_0}\,\frac{q1q2}{{r_{21}}^2}\,\widehat{\boldsymbol{r_{21}}}}$; $\mathbf{F}_{21}$ → Force on q_2 due to q_1, $\boldsymbol{r_{21}} \rightarrow \boldsymbol{r_2} - \boldsymbol{r_1}$; $\boldsymbol{r_1}$ & $\boldsymbol{r_2}$ are the position vectors of q_1 and q_2 and $\widehat{\boldsymbol{r_{21}}}$ → Unit vector in the direction of $\boldsymbol{r_{21}}$.

4. Write the pictorial representations of the force of repulsion and attraction, between two point charges.

A: (1) For two like charges:



(2) For two unlike charges:



5. Explain the principle of superposition to calculate the force between multiple charges.

A: The principle of superposition:

It is the principle which gives a method to find the force on a given charge due a group of charges interacting with it. According to this principle "force on any charge due to a number of other charges is the vector sum of all the forces on that charge due to the other charges, taken one at a time. The individual forces are unaffected due to the presence of other charges".

To understand this concept, consider a system of charges q_1, q_2.........q_n. The force on q_1 due to q_2 is being unaffected by the presence of the other charges q_3, q_4.........q_n. The total force F_1 on the charge q_1 due to all other charges is then given by superposition principle

$$\mathbf{F}_1 = \mathbf{F}_{12} + \mathbf{F}_{13} + \ldots + \mathbf{F}_{1n} = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1 q_2}{r_{12}^2}\hat{\mathbf{r}}_{12} + \frac{q_1 q_3}{r_{13}^2}\hat{\mathbf{r}}_{13} + \ldots + \frac{q_1 q_n}{r_{1n}^2}\hat{\mathbf{r}}_{1n}\right]$$

$$= \frac{q_1}{4\pi\varepsilon_0}\sum_{i=2}^{n}\frac{q_i}{r_{1i}^2}\hat{\mathbf{r}}_{1i}$$

6. Mention the expression for the electric field due to a point charge placed in vacuum.

A: Electric field , $\boxed{\mathbf{E} = \frac{1}{4\pi\varepsilon_0}\ \frac{q}{r^2}\ \hat{\boldsymbol{r}}}$

7. Write the expression for the electric field due to a system of charges and explain it.

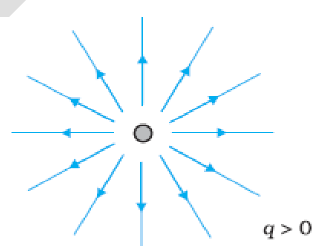
A: Electric field due to a system of charges q_1, q_2, q_3… ,q_n described by the position vectors r_1 ,r_2 ,r_3 ,………………,r_n respectively relative to some origin. Using Coulomb's law and the principle of superposition, it can be shown that the electric field **E** at a point P represented by the position vector **r**, is given by

$$\mathbf{E(r)} = \frac{1}{4\pi\varepsilon_0}\left\{\frac{q1}{r1p^2}\,\widehat{\boldsymbol{r}_{1P}} + \frac{q2}{r2p^2}\,\widehat{\boldsymbol{r}_{2P}} + \frac{q3}{r3p^2}\,\widehat{\boldsymbol{r}_{3P}} + \ldots\ldots\ldots\ldots\ldots + \frac{qn}{rnp^2}\,\widehat{\boldsymbol{r}_{nP}}\right\}$$

Or $\boxed{\mathbf{E(r)} = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{i=n}\frac{qi}{rip^2}\widehat{\boldsymbol{r}_{iP}}}$

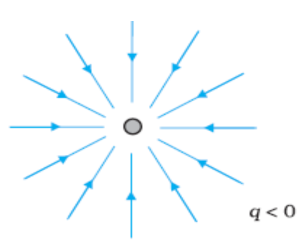
8. Draw electric field lines in case of a positive point charge.

A:



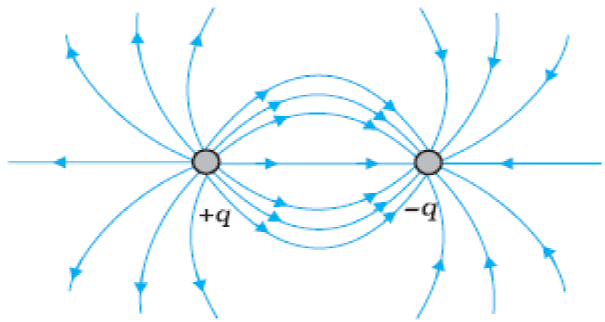
9. Sketch electric field lines in case of a negative point charge.

A:



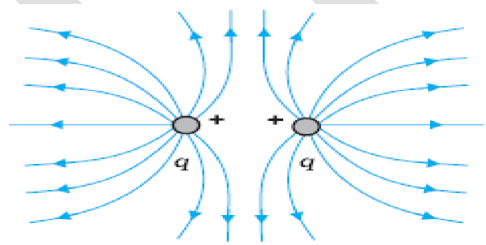
10. Sketch the electric field lines in case of an electric dipole.

A:



11. Sketch the electric field lines in case of two equal positive point charges.

A:



12. Mention any two properties of electric field lines.

A: (1) Electric field lines start from a positive charge and end on a negative charge.

(2) Electric field lines do not intersect each other.

13. Write the expression for the torque acting on an electric dipole placed in a uniform electric field and explain the terms in it.

A: τ = PE sinθ; $\tau \rightarrow$torque, P → dipole moment of the electric dipole, E → Strength of the uniform electric field and θ → angle between the directions of P and E.

14. Define linear density of charge and mention its SI unit.

A: Linear density of charge is charge per unit length. Its SI unit is Cm^{-1}

15. Define surface density of charge and mention its SI unit.

A: Surface density of charge is charge per unit area. Its SI unit is Cm^{-2}

16. Define volume density of charge and mention its SI unit.

A: Volume density of charge is charge per unit volume. Its SI unit is Cm^{-3}

17. What is the effect of a non-uniform electric field on an electric dipole?

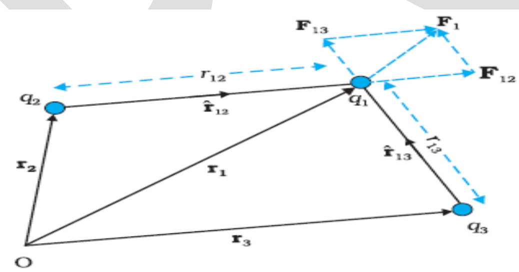
A: In a non-uniform electric field an electric dipole experiences both the torque and a net force.

Three mark questions with answers

1. Mention three properties of electric charge.

A: 1.Electrc charge is conserved   2. Electric charge is quantised 3. Electric charge is additive

2. Draw a diagram to show the resultant force on a charge in a system of three charges.
A:



3. Why is the electric field inside a uniformly charged spherical shell, zero? Explain.

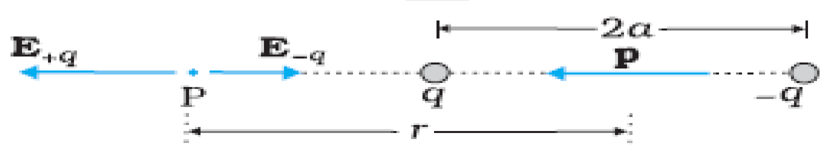
A: When a spherical shell is charged, the charges get distributed uniformly over its outer surface and the charge inside the shell is zero. According to Gauss's law, as the charge inside is zero, the electric flux at any point inside the shell will be zero. Obviously the electric field (electric flux per unit area) is also zero.

Five mark questions with answers

1. Obtain an expression for the electric field at a point along the axis of an electric dipole.

A: Consider a point 'P' on the axis of an electric dipole at a distance 'r' from its mid-point as shown in the figure. The magnitude of dipole moment of the dipole '**p**' (directed from –q to +q), is given by

$$|\mathbf{p}| = q \times 2a \quad \ldots\ldots(1)$$



The electric field at P due to –q is $\mathbf{E}_{-q} = -\dfrac{q}{4\pi\varepsilon_0(r+a)^2}\,\hat{\mathbf{p}}$ towards –q

The electric field at P due to +q is $\mathbf{E}_{+q} = +\dfrac{q}{4\pi\varepsilon_0(r-a)^2}\,\hat{\mathbf{p}}$ away from +q;

$\hat{\mathbf{p}}$ is a unit vector in the direction of '**p**', (from –q to +q)

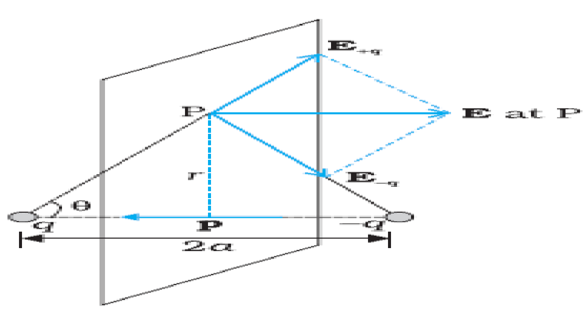
According to the principle of superposition, the total electric field at P, '**E**' is given by

$$\mathbf{E} = \mathbf{E}_{+q} + \mathbf{E}_{-q} = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2}\right] = \frac{q}{4\pi\varepsilon_0}\frac{4ar}{(r^2-a^2)^2}\,\hat{\mathbf{p}}$$

For **r >> a**, $\mathbf{E} = \dfrac{q}{4\pi\varepsilon_0}\left[\dfrac{4a}{r^3}\right]\hat{\mathbf{p}}$ or $\boxed{\mathbf{E} = \dfrac{1}{4\pi\varepsilon_0}\left[\dfrac{2p}{r^3}\right]\hat{\mathbf{p}}}$

2. Obtain an expression for the electric field at a point on the equatorial plane of an electric dipole.

A:



Consider a point ‘P’ on the perpendicular bisector of the axis of an electric dipole at a distance ‘r’ from its mid-point as shown in the figure. The dipole moment of the dipole ‘**p**’ (directed from –q to +q), whose magnitude is given by

$$|\mathbf{p}| = q\text{x}2a\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

The electric field at P due to –q is $\mathbf{E}_{-q} = \dfrac{q}{4\pi\,\varepsilon_0\,(r^2+a^2)}\,\hat{\mathbf{p}}$ towards –q

The electric field at P due to +q is $\mathbf{E}_{+q} = \dfrac{q}{4\pi\varepsilon_0(r^2+a^2)}\,\hat{\mathbf{p}}$ away from +q;

Hence $\mathbf{E}_{-q} = \mathbf{E}_{+q}$ **(in magnitude)**

Resolving these two fields into two rectangular components each, we see that the components perpendicular to the dipole axis cancel each other and the components parallel to the dipole axis add up. Therefore the total electric field at P is given by

$$\mathbf{E} = -(\mathbf{E}_{+q} + \mathbf{E}_{-q})\cos\theta\ \hat{\mathbf{p}}$$

$$= -\left\{\frac{q}{4\pi\,\varepsilon_0\,(r^2+a^2)} + \frac{q}{4\pi\varepsilon_0(r^2+a^2)}\right\}\left[\frac{a}{\sqrt{(r^2+a^2)}}\right]\hat{\mathbf{p}}$$

$$= -\frac{2qa}{4\pi\varepsilon_0(r^2+a^2)^{\frac{3}{2}}}\,\hat{\mathbf{p}} = -\frac{\mathrm{p}}{4\pi\varepsilon_0(r^2+a^2)^{\frac{3}{2}}}\,\hat{\mathbf{p}} \quad \{\text{from (1)}\}$$

For **r >> a**,

$$\boxed{\mathbf{E} = -\frac{1}{4\pi\varepsilon_0}\left[\frac{\mathrm{p}}{r^3}\right]\hat{\mathbf{p}}}$$

(Negative sign implies that the direction of **E** is anti-parallel to the direction of **p)**

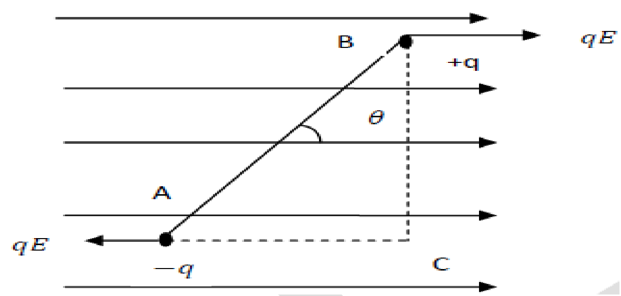
3. Obtain the expression for the torque acting on an electric dipole placed in a uniform electric field.

A: Consider an electric dipole consisting of charges $-q$ &$+q$ and of dipole length 2a placed in a uniform electric field E making an angle θ with the direction of E.The force acting on +q is qE in the direction of E and the force acting on –q is also qE but in a direction opposite to the direction of E. Hence these two equal and parallel forces constitute a couple and torque experienced by the dipole is given by,

τ = force x perpendicular distance between the two forces (called ARM of the couple)

i.e., $\tau = (qE)\,(BC) = qE\,(2a\,sin\theta) = pE\,sin\theta$ $i.e. \tau = pE\sin\theta$

This torque on dipole always tends to align the dipole in the direction of the electric field and is normal to the plane containing **p** and ***E***



4. Using Gauss's law, obtain an expression for the electric field due to an infinitely long straight uniformly charged conductor.

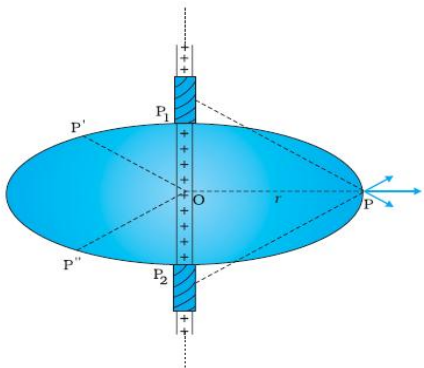
A: Consider an infinitely long thin straight wire with uniform linear charge density 'λ'. The wire is obviously an axis of symmetry. Suppose we take the radial vector from O to P and rotate it around the wire. The points P, P', P'' so obtained are completely equivalent with respect to the charged wire. This implies that the electric field must have the same magnitude at these points. The direction of electric field at every point must be radial (outward if $\lambda > 0$, inward if $\lambda < 0$). This is clear from Fig1.
Consider a pair of line elements P_1 and P_2 of the wire, as shown. The electric fields produced by the two elements of the pair when summed gives the resultant electric field which is radial (the components normal to the radial vector cancel). This is true for any such pair and hence the total field at any point P is radial. Finally, since the wire is infinite, electric field does not depend on the position of P along the length of the wire. In short, the electric field is everywhere radial in the plane cutting the wire normally, and its magnitude depends only on the radial distance *r*.
To calculate the field, imagine a cylindrical Gaussian surface, as shown in the Fig.2 Since the field is everywhere radial, flux through the two ends of the cylindrical Gaussian surface is zero. At the cylindrical part of the surface, **E** is normal to the surface at every point, and its magnitude is constant, since it depends only on r. The surface area of the curved part is $2\pi rl$, where l is the length of the cylinder.
Flux through the Gaussian surface = flux through the curved cylindrical part of the surface

$= E \times 2\pi rl$

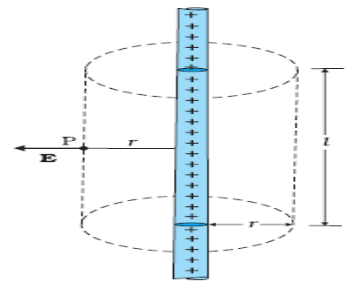
The surface includes charge equal to λl. Gauss's law then gives $E \times 2\pi rl = \lambda l/\varepsilon_0$

i.e., $E = \lambda/2\pi\varepsilon_0 r$

Vectorially, **E** at any point is given by **E** = $\frac{\lambda}{2\pi\ \varepsilon 0 r}\,\widehat{\mathbf{n}}$; $\widehat{\mathbf{n}}$ is the radial unit vector in the plane, normal to the wire passing through the point. **E** is directed outward if λ is positive and inward if λ is negative.



fig(1)



fig(2)

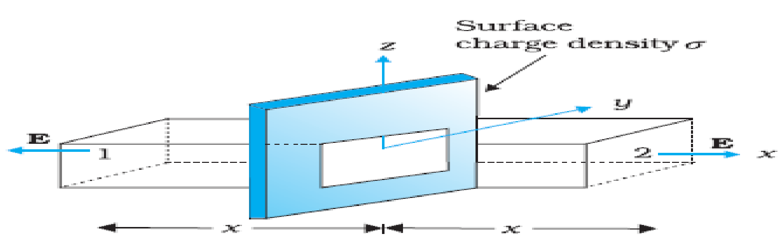
5. Using Gauss's law, obtain an expression for the electric field due to a uniformly charged infinite plane sheet.

A: Let σ be the uniform surface charge density of an infinite plane sheet. We take the *x*-axis normal to the given plane. By symmetry, the electric field will not depend on *y* and *z* coordinates and its direction at every point must be parallel to the *x*-direction.

We can take the Gaussian surface to be a rectangular parallelepiped of cross sectional area 'A'. As seen from the figure, only the two faces 1 and 2 will contribute to the flux; electric field lines are parallel to the other faces and they, therefore, do not contribute to the total flux. The unit vector normal to surface 1 is in –*x* direction while the unit vector normal to surface 2 is in the +*x* direction. Therefore, flux **E**.Δ**S** through both the surfaces is equal and add up. Therefore the net flux through the Gaussian surface is *2* EA. The charge enclosed by the closed surface is σA.

Therefore by Gauss's law, 2 EA = $\sigma A/\varepsilon_0$ or, E = $\sigma/2\varepsilon_0$

Vectorially, $\boxed{\mathbf{E} = \sigma/2\varepsilon_0\,\widehat{\mathbf{n}}}$; $\widehat{\mathbf{n}}$ is a unit vector normal to the plane and going away from it. **E** is directed away from the plate if σ is positive and toward the plate if σ is negative.



6. Using Gauss's law, obtain an expression for the electric field at an outside point due to a uniformly charged thin spherical shell.

A: Let σ be the uniform surface charge density of a thin spherical shell of radius *R*. The situation has obvious spherical symmetry. The field at any point P, outside or inside, can depend only on *r* (the radial distance from the centre of the shell to the point) and must be radial (i.e., along the radius vector).

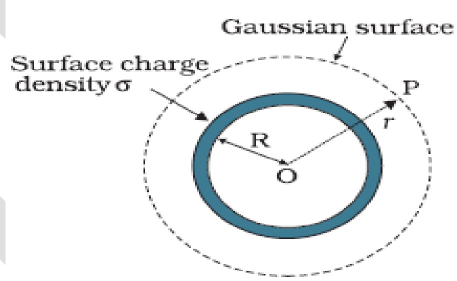
Consider a point P outside the shell with radius vector **r**. To calculate **E** at P, we take the Gaussian surface to be a sphere of radius *r* and with centre O, passing through P. All points on this sphere are equivalent relative to the given charged configuration. (That is what we mean by spherical symmetry.) The electric field at each point of the Gaussian surface, therefore, has the same magnitude *E* and is along the radius vector at each point. Thus, **E** and Δ**S** at every point are parallel and the flux through each element is E ΔS. Summing over all Δ*S*, (i) the flux through the Gaussian surface is $E \times 4\pi r^2$ and (ii) the charge enclosed is $\sigma \times 4\pi R^2$.

By Gauss's law, $E \times 4\pi r^2 = \frac{\sigma}{\varepsilon 0} \times 4\pi R^2$

Or $E = \frac{\sigma R^2}{\varepsilon 0 r^2} = \frac{q}{4\pi\varepsilon_0 r^2}$; where $q = (4\pi R^2 \times \sigma)$ is the total charge on the spherical shell.

Vectorially, $\boxed{\mathbf{E} = \frac{q}{4\pi\varepsilon_0 r^2}\,\hat{\mathbf{r}}}$; $\hat{\mathbf{r}}$ is the unit vector in the direction of **E**

(The electric field is directed outward if $q > 0$ and inward if $q < 0$).



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