GOVERNMENT OF KARNATAKA

KARNATAKA STATE PRE-UNIVERSITY EDUCATION EXAMINATION BOARD

II YEAR PUC EXAMINATION

SCHEME OF VALUATION (OS)

Subject Code : 35 Subject : MATHEMATICS

Qn. No.	PART-A	Marks
1.	$a\mid b \Rightarrow b=ak \Rightarrow bx=akx \Rightarrow bx=ak \therefore a\mid bx$	1
2.	$\lvert AB\rvert = \lvert A\rvert.\lvert B\rvert = (8)(10) = 80$	1
3.	$a*e=a \Rightarrow \frac{ae}{13}=a \quad \therefore e=13$	1
4.	P.V. of mid p.t of $\vec{AB} = 2\hat{i}-\hat{j}+2\hat{k}$	1
5.	$h=0$	1
6.	Parabola	1
7.	$\sin^{-1}(\sin(-10^\circ)) = -10^\circ$	1
8.	$(1-\omega+\omega^2)^6 = (-2\omega)^6 = 64$	1
9.	$\frac{dy}{dx} = x\cdot\left(\frac{1}{x}\right)+\log x(1) = 1+\log x$	1
10.	$\int_0^{\pi/4}\sec^2 x\,dx = [\tan x]_0^{\frac{\pi}{4}} = 1-0=1$	1

Qn. No.	PART-B.	Marks
11.	$p \nmid a,\ (a, p) = 1 \Rightarrow ax + py = 1,\ x, y \in I$	
	$\Rightarrow abx + pby = b$	1
	$p \mid ab \Rightarrow ab = pk \therefore pkx + pby = b$	
	$\therefore p(K) = b \Rightarrow p \mid b$	1
12.	$\Delta = 7 \qquad \Delta_1 = -14 \quad \therefore x = -2$	1
	$\Delta_2 = 21 \qquad y = 3$	1
	(If $\Delta, \Delta_1, \Delta_2$ are correct award 1 mark)	
13.	Let e & e' be two identity elements	
	then $e * e' = e' * e = e'$ &	
	$e' * e = e * e' = e$	1
	$\Rightarrow \quad e = e' \therefore$ Identity element is unique	1
14.	$(5\vec{a} - \vec{b}) \times (\vec{a} + 3\vec{b})$	
	$= 5(\vec{a} \times \vec{a}) + 15(\vec{a} \times \vec{b}) - (\vec{b} \times \vec{a}) - 3(\vec{b} \times \vec{b})$	1
	$= 16(\vec{a} \times \vec{b})$	1
15.	$C_1(2, 3) \quad C_2(-3, -9),\ r_1 = 5,\ r_2 = 8$	1
	$C_1C_2 = r_1 + r_2 = 13$	1
16.	$(x-4)^2 = -2\left(y - \frac{9}{2}\right)$	1
	Vertex $(h.k) = \left(4, \frac{9}{2}\right)$	1
17.	$\tan^{-1}\left(\frac{x+y}{1-xy}\right) = \frac{\pi}{4}$	1
	$\frac{x+y}{1-xy} = 1 \quad \therefore x + y + xy = 1$	1

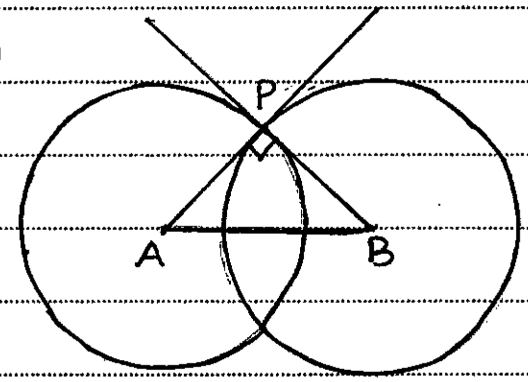
Qn. No.		Marks
18	Writing $x^2-2x\cos\theta+1=0$	1
	Getting $x=\frac{2\cos\theta \pm 2\sqrt{\cos^2\theta-1}}{2}$, $x=e^{i\theta}$	1
19.	$\log y = \sin x \log x$	1
	$\frac{dy}{dx} = x^{\sin x}\left(\frac{\sin x}{x}+\log x \cos x\right)$	1
20.	$\left(\frac{dy}{dx}\right)_{(4,3)} = \frac{1}{3}$	1
	Sub tangent (S.T) $= \frac{y_1}{f'(x_1)} = \frac{3}{\left(\frac{1}{3}\right)} = 9$	1
21.	Writing $\int \frac{e^x(x+1-1)}{(x+1)^2}\,dx$	1
	Getting $\frac{e^x}{x+1}+c$	1
22.	Writing $x^2+y^2=r^2$	1
	Getting $\frac{dy}{dx}=-\frac{x}{y}$	1

Qn. No.	PART- C	Marks
I. 23	Division	1
	Writing $(275 \quad 726) = 11$	1
	$11 = 4 \times 77 - 3 \times 99$	1
	$11 = 726\,(11) - 29 \times 275$	
	OR	
	$11 = 275(-29) + 726(+11)$ $\quad x = -29$, $y = 11$	1
	Getting $x' = 697 \quad y' = -264$	
	OR	1
	$x' = -755 \quad y' = 286$	
24.	$X = A^{-1}B$ & $A^{-1} = \dfrac{\text{Adj}A}{\lvert A \rvert}$	1
	$\lvert A \rvert = 93$	1
	Co-factor matrix $\begin{bmatrix} 60 & -27 & -37 \\ -6 & 12 & 13 \\ 3 & -6 & 9 \end{bmatrix}$	1
	(any four correct co-factors. give one mark)	
	Adjoint of $A = \begin{bmatrix} 60 & -27 & -37 \\ -6 & 12 & 13 \\ 3 & -6 & 9 \end{bmatrix}$	1
	Getting $x = 1, \quad y = -1, \quad z = 2$	1

Qn. No.		Marks
25(a)		

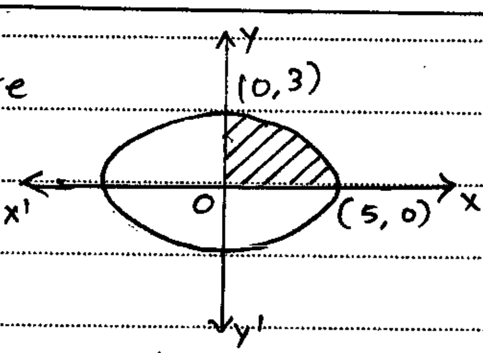
$\times_{10}$	2	4	6	8
2	4	8	2	6
4	8	6	4	2
6	2	4	6	8
8	8	6	4	2

Qn. No.		Marks
	(table)	1
	Identity $e = 6$	1
	$2 \times_{10} 4^{-1} = 2 \times_{10} 4 = 8$	1
(b)	$(ab)(ab) = (aa)(bb)$ $\Rightarrow a[b(ab)] = a[a(bb)]$	1
	$\Rightarrow b(ab) = a(bb)$ $\Rightarrow (ba)b = (ab)b$ $ab = ba \therefore$ G is an abelian	1
26(a)	Finding $\vec{a} \times (\vec{b} \times \vec{c}) = -7\hat{i} - 3\hat{j} + \hat{k}$	1
	Writing $\hat{n} = \pm \dfrac{\vec{a} \times (\vec{b} \times \vec{c})}{\lvert \vec{a} \times (\vec{b} \times \vec{c}) \rvert}$	1
	Getting $\hat{n} = \pm \left(\dfrac{-7\hat{i} - 3\hat{j} + \hat{k}}{\sqrt{59}} \right)$	1

Qn. No.		Marks
26(b)	$\vec{a}\cdot(\vec{b}\times\vec{c}) = \begin{vmatrix} 3 & 1 & -2 \\ 1 & 4 & 1 \\ 2 & -1 & 5 \end{vmatrix}$	1
	Volume = 78 cubic units	1
II.27(a)	 Correct fig	1
	For writing $AB^2 = AP^2 + PB^2$ $A \equiv (-g_1, -f_1)$ $B \equiv (-g_2, -f_2)$ $AP = \sqrt{g_1^2+f_1^2-c_1}$ $BP = \sqrt{g_2^2+f_2^2-c_2}$	1
	Getting $2g_1g_2 + 2f_1f_2 = c_1 + c_2$	1
	[No figure or wrong fig carries zero mark]	
b.	R. Axes of circles $2x + 8y - 6 = 0$ — (1) $8x - 4y - 24 = 0$ — (2) $10x + 4y - 30 = 0$ — (3)	1
	Radical centre $(3, 0)$	1
	(Any two R. Axes are correct award 1 mark)	

Qn. No.		Marks
28(a)	Writing the equation $(y-k)^2 = 4a(x-h)$	1
	Getting $a = -2$	1
	Writing $(y-3)^2 = -8(x-4)$	1
(b)	$\frac{2a}{e} = 10\sqrt{2}$ & $e = \frac{1}{\sqrt{2}}$. Getting $a = 5$	1
	Getting $b = \frac{5}{\sqrt{2}}$	
	Writing the eq^n $\frac{x^2}{25} + \frac{y^2}{\left(\frac{25}{2}\right)} = 1$	1
29(a)	$\cos^{-1}x = \alpha$ $\quad \cos^{-1}y = \beta$ $\quad \cos^{-1}z = \gamma$	
	$\cos\alpha = x$ $\quad \cos\beta = y$ $\quad \cos\gamma = z$	1
	$\sin\alpha = \sqrt{1-x^2}$ $\quad \sin\beta = \sqrt{1-y^2}$ $\quad \sin\gamma = \sqrt{1-z^2}$	
	$\cos(\alpha+\beta) = \cos(\pi-\gamma)$	1
	$xy - \sqrt{1-x^2}\sqrt{1-y^2} = -z$	
	Getting $x^2+y^2+z^2+2xyz = 1$	1
	OR	
	$\cos^{-1}\left(xy - \sqrt{1-x^2}\sqrt{1-y^2}\right) = \pi - \cos^{-1}x$	1
	$xy - \sqrt{1-x^2}\sqrt{1-y^2} = -z$	1
	Getting $x^2+y^2+z^2+2xyz = 1$	1
(b)	$\sin^2\theta - 1 + 2\sin^2\theta = \frac{5}{4} \Rightarrow \sin\theta = \pm\frac{\sqrt{3}}{2}$	1
	$\theta = \pm\left(\frac{\pi}{3}\right)$, $\theta = n\pi + (-1)^n\left(\pm\frac{\pi}{3}\right)$, $n \in I$	1
	OR	
	(Any alternative method award proportinate marks)	

Qn. No.		Marks
III 30(a)	$\frac{dy}{dx} = \lim_{\delta x \to 0}\left[\frac{f(x+\delta x)-f(x)}{\delta x}\right]$	1
	$= \lim_{\delta x \to 0}\left[\frac{1}{x}\log_e\left(1+\frac{\delta x}{x}\right)^{x/\delta x}\right]$	1
	$\frac{dy}{dx} = \frac{1}{x}$	1
(b)	Writing $y = \tan^{-1}3 - \tan^{-1}2x$	1
	$\frac{dy}{dx} = \frac{-2}{1+4x^2}$	1
31(a)	$y_1 = 5(x+\sqrt{x^2+1})^4\left(1+\frac{1}{\cancel{2}\sqrt{x^2+1}}(\cancel{2}x)\right)$	1
	Getting $(x^2+1)2y_1y_2+2xy_1^2 = 50yy_1$	1
	Simplifying & getting $(x^2+1)y_2+xy_1-25y=0$	1
(b)	Getting $\frac{1}{2\sqrt{x}}+\frac{1}{2\sqrt{y}}\frac{dy}{dx}=0$	1
	Slope of tangent $\left(\frac{dy}{dx}\right)_{\left(\frac{1}{4},\frac{1}{4}\right)} = -1$	1
32(a)	Substituting $x = \cos\theta$	1
	Getting $y = \frac{1-x}{2}$	1
	Getting $\frac{dy}{dx} = -\frac{1}{2}$	1
	OR (Any alternative method award proportinate mark)	

Qn. No.		Marks
32(b)	$\sqrt{x^2-4x+2} = \sqrt{(x-2)^2-(\sqrt{2})^2}$	1
	Getting $I = \cosh^{-1}\left(\frac{x-2}{\sqrt{2}}\right)+c.$	1
33(a)	$\tan\left(\frac{x}{2}\right)=t$ $dx=\frac{2dt}{1+t^2}$ and $\cos x=\frac{1-t^2}{1+t^2}$	1
	Getting $I = 2\int\frac{dt}{(3)^2+t^2}$	1
	$I = \frac{2}{3}\tan^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)}{3}\right)+c.$	1
33(b)	Writing $\int\frac{x^2}{x^3\sqrt{(x^3)^2-1}}\,dx$ & Put $x^3=t.$	1
	Getting $I = \frac{1}{3}\sec^{-1}(x^3)+C.$	1
34.	Correct figure 	1
	Getting $Y=\frac{3}{5}\sqrt{25-x^2}$	1
	Area $= 4\times\frac{3}{5}\int_0^5\sqrt{25-x^2}\,dx$	1
	Correct integration	1
	Area $= 15\pi$ sq. units.	1
	(Without fig, with correct answer deduct 1 mark)	

Qn. No.		Marks
	OR	
	(Proving the area of ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is πab. Put $a=5$ $b=3$ Area 15π sq units. award full marks)	
	PART-D	
35(a)	Definition as locus of a point only	1
	Correct figure	1
	Getting $CS = ae$	1
	and $CZ = \frac{a}{e}$	1
	Writing $PS = ePM$ $(e>1)$	1
	Substituting for PS and PM and Simplifying to get $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$. where $b^2=a^2(e^2-1)$	1
	[No figure or wrong figure carries zero mark]	
(b)	By $R_1' = R_1+R_2+R_3$	
	L.H.S $= \begin{vmatrix} a+b+c & a+b+c & a+b+c \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$	1
	$=(a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$	1
	$C_2' = C_2 - C_1$ & $C_3' = C_3 - C_1$	
	$=(a+b+c)\begin{vmatrix} 1 & 0 & 0 \\ 2b & -(a+b+c) & 0 \\ 2c & 0 & -(a+b+c) \end{vmatrix}$	1
	Rest	1

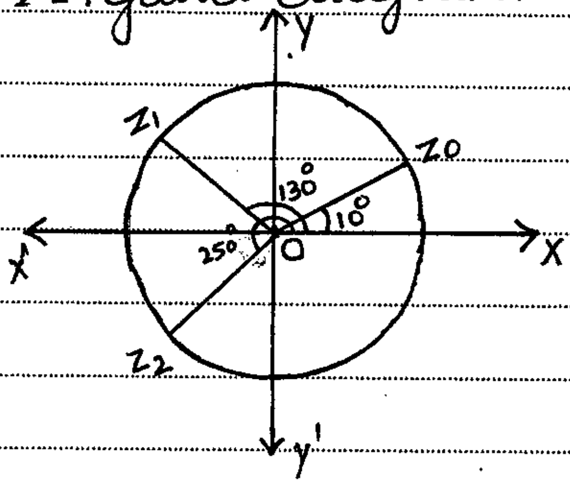
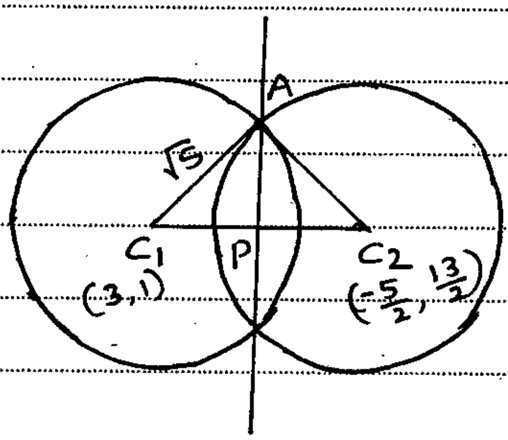
Qn. No.		Marks
36(a)	Statement: If n is an integer	
	$(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$	
	and if n is a fraction then one of	
	the values of $(\cos\theta+i\sin\theta)^n$ is $\cos n\theta + i\sin n\theta$	1
	Proving for positive integers	2
	Proving for negative integers	2
	Proving for fractions	1
(b)	Writing $\cos x - \sqrt{3}\sin x = -\sqrt{2}$	1
	Getting $\cos x\cos(\pi/3) - \sin x\sin(\frac{\pi}{3}) = \frac{-1}{\sqrt{2}}$	1
	$\cos\left(x+\frac{\pi}{3}\right) = \cos\left(\frac{3\pi}{4}\right)$	1
	Getting $x = 2n\pi \pm \frac{3\pi}{4} - \frac{\pi}{3}$	1
	OR	
	Writing $\sqrt{3}\sin x - \cos x = \sqrt{2}$	1
	Getting $\sin x\cos\left(\frac{\pi}{6}\right) - \cos x\sin\left(\frac{\pi}{6}\right) = \frac{1}{\sqrt{2}}$	1
	$\sin\left(x - \frac{\pi}{6}\right) = \sin\left(\frac{\pi}{4}\right)$	1
	Getting $x = n\pi + (-1)^n\frac{\pi}{4} + \frac{\pi}{6}$	1
37(a)	$V = \frac{4}{3}\pi r^3$ and $S = 4\pi r^2$	1
	Getting $r = 5$	1
	$\frac{ds}{dt} = 4\pi\cdot 2r\left(\frac{dr}{dt}\right)$	1
	Getting $\frac{dr}{dt} = \frac{1}{5\pi}$	1
	$\frac{dv}{dt} = \frac{4}{3}\pi\cdot 3r^2\left(\frac{dr}{dt}\right)$	1
	$\frac{dv}{dt} = 20$ cc/sec	1

Qn. No.		Marks
37(b)	Statement	1
	$\lvert A-\lambda I\rvert=0 \Rightarrow \lambda^2-5\lambda-22=0$	1
	By. C.H.Thm $A^2-5A-22I=0$	1
	Getting $A^{-1} = -\frac{1}{22}\begin{bmatrix}3 & -4\\ -7 & 2\end{bmatrix}$	1
	OR $A^{-1} = \frac{1}{22}\begin{bmatrix}-3 & 4\\ 7 & -2\end{bmatrix}$	
	[If I is not written deduct 1 mark]	
38(a)	$I = \int_0^{\pi} \frac{\pi - x}{1+\sin^2 x}\,dx$	1
	$2I = \pi \int_0^{\pi} \frac{1}{1+\sin^2 x}\,dx$	1
	$= \pi \cdot 2 \int_0^{\frac{\pi}{2}} \frac{1}{1+\sin^2 x}\,dx$	1
	$= 2\pi \int_0^{\frac{\pi}{2}} \frac{\sec^2 x}{\sec^2 x + \tan^2 x}\,dx$ (Divide Nr & Dr by $\cos^2 x$)	1
	$= 2\pi \int_0^{\pi} \frac{\sec^2 x}{1+2\tan^2 x}\,dx$	
	$= 2\pi \int_0^{\infty} \frac{dt}{1+2t^2}$	1
	Getting $I = \frac{\pi^2}{2\sqrt{2}}$	1

Qn. No.		Marks
38(b)	Put $x-y=z \Rightarrow \frac{dy}{dx} = 1 - \frac{dz}{dx}$	1
	Getting $\frac{z^2}{z^2-a^2}\,dz = dx$	1
	$\int\left(\frac{z^2-a^2+a^2}{z^2-a^2}\right)dz = \int dx$	
	Writing $\int\left(1+\frac{a^2}{z^2-a^2}\right)dz = \int dx$	1
	Writing $(x-y)+\frac{a}{2}\log\left[\frac{(x-y)-a}{(x-y)+a}\right] = x+c$	1
	[Without c deduct one mark]	

PART-E

Qn. No.		Marks
39(a)	$r = 2\sqrt{3}$ and $\theta = \frac{\pi}{6}$	1
	$3+\sqrt{3}i = 2\sqrt{3}\,CiS\left[\frac{\pi}{6}\right]$	
	$= 2\sqrt{3}\,CiS\left(2n\pi + \frac{\pi}{6}\right)$	
	$(3+\sqrt{3}i)^{\frac{1}{3}} = (2\sqrt{3})^{\frac{1}{3}}\,CiS\left(\frac{12n\pi+\pi}{18}\right)$	1
	Writing $z_0 = (2\sqrt{3})^{\frac{1}{3}}\,CiS\left(\frac{\pi}{18}\right)$	
	$z_1 = (2\sqrt{3})^{\frac{1}{3}}\,CiS\left(\frac{13\pi}{18}\right)$	1
	$z_2 = (2\sqrt{3})^{\frac{1}{3}}\,CiS\left(\frac{25\pi}{18}\right)$	

Qn. No.		Marks
	Argand diagram.	1
		
39 (b)	Figure	1
		
	Getting radical axis is $-11x+11y-11=0$ or $x-y+1=0$	1
	$C_1P = \left\lvert \dfrac{ax_1+by_1+c}{\sqrt{a^2+b^2}} \right\rvert = \dfrac{3}{\sqrt{2}}$	1
	$AP = \sqrt{5-\frac{9}{2}} = \sqrt{\frac{1}{2}}$	
	$\therefore$ Length of chord $AB = 2\left(\frac{1}{\sqrt{2}}\right)$ or $\sqrt{2}$ units	1

Qn. No.		Marks
39(c)	$(2^3)^{51} \equiv 1 \pmod 7$	1
	Multiply by 2^2 and	
	Getting $2^{155} \equiv 4 \pmod 7$	1
	$\therefore$ Remainder is 4	
40(a)	$\lvert\vec{a}+\vec{b}\rvert^2 + \lvert\vec{a}-\vec{b}\rvert^2 = 2(\lvert\vec{a}\rvert^2+\lvert\vec{b}\rvert^2)$	1
	$\lvert\vec{a}+\vec{b}\rvert^2 = 2(\lvert\vec{a}\rvert^2+\lvert\vec{b}\rvert^2) - \lvert\vec{a}-\vec{b}\rvert^2$	1
	$= 2(169+361) - 484$	1
	Getting $\lvert\vec{a}+\vec{b}\rvert = 24$	1
(b)	Writing $\int \left(\frac{1+\cos 2x}{2}\right)^2 dx$	1
	$= \int \frac{1}{4}\left[1+\cos^2 2x + 2\cos 2x\right] dx$	
	$= \frac{1}{4}\int \left[1 + \frac{1+\cos 4x}{2} + 2\cos 2x\right] dx$	1
	$= \frac{1}{8}\int (3 + \cos 4x + 4\cos 2x)\, dx$	1
	$= \frac{1}{8}\left[3x + \frac{\sin 4x}{4} + 4\frac{\sin 2x}{2}\right] + c$	1

Qn. No.		Marks
40		
(c)	$y = \tan\left(\frac{5\pi x}{180}\right)$	1
	$\frac{dy}{dx} = \sec^2\left(\frac{5\pi x}{180}\right)\left(\frac{5\pi}{180}\right)$	1
	OR	
	$y = \tan\left(\frac{\pi x}{36}\right)$	1
	$\frac{dy}{dx} = \sec^2\left(\frac{\pi x}{36}\right)\left(\frac{\pi}{36}\right)$	1
	——— x x x x ———	